The role of the radius in the Geometric Ratio Model (GRM)

From bounding square to derived radius, a practical reformulation of circular geometry

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Abstract

This whitepaper explores the role of the radius within the Geometric Ratio Model (GRM), a system that redefines geometric interpretation based on proportional structure rather than classical abstraction. In GRM, the radius is not a given parameter but a derived quantity; logically inferred from the bounding square or cube in which a shape is inscribed. The paper introduces the concept of the radius as a system constant (0.1250 SPU), outlines its behavior across dimensions (1D, 2D, 3D), and demonstrates how this allows for ratio-based classification and fuzzy interpretation of near-circular forms. The derived radius enables a structural transition from classical geometry to a digital geometry paradigm, supporting real-time recognition, deviation analysis, and dimensionally consistent modeling in AI, CAD, and image processing applications. The result is a shift from fixed measurements to proportional logic, making the radius not only a geometric quantity, but a key to understanding and interpreting visual form in digital systems.

Peer Review and Scientific Dialogue

This whitepaper is part of the GRM Foundation Series and represents version 2.0 of *The Role of the Radius in the Geometric Ratio Model*. It builds upon earlier insights, now expanded with structural derivations, dimensional scaling, fuzzy classification, and digital system implications.

The content has been internally reviewed for consistency and completeness. However, as the GRM is an emerging model, **external peer review is actively invited** to ensure academic rigor, to challenge assumptions, and to explore potential extensions or applications.

We welcome feedback, questions, or formal reviews from scholars, engineers, educators, and practitioners in the following domains:

- Geometry and mathematical modeling
- Digital image analysis and computer vision
- CAD and design engineering
- AI and machine reasoning
- Didactics and mathematical education

Peer reviewers are encouraged to assess:

- Internal consistency of the GRM framework
- Validity of the dimensional ratios and derivations
- Clarity of the logic connecting radius, proportion, and identity
- Applicability of the GRM in visual or computational systems

Comments or review proposals can be submitted via:
info@inratios.com or through the GRM knowledge portal at www.inratios.com

All serious contributions will be acknowledged in future revisions or companion publications, subject to permission.

Version Notes

This document constitutes version 2.0 of the whitepaper *The Role of the Radius in the Geometric Ratio Model*. Compared to version 1.1, this revised edition offers:

- A completely restructured narrative, moving from classical geometry toward GRM in a didactic progression;
- Explicit formulation of the radius as a system-derived constant (0.1250 SPU);
- Integration of dimensionally consistent scaling logic ($r^1 \rightarrow r^2 \rightarrow r^3$);
- Introduction of fuzzy classification and proportional identity logic;
- A new chapter on practical implications in AI, CAD, and visual systems;
- Expanded appendices with ratio tables, derivations, and visual references.

The goal of this revision is to strengthen both theoretical clarity and practical applicability, while aligning with the broader GRM Foundation Series.

Inhoud

1. Introduction – From Radius to Ratio

1.1 Classical Geometry: The Radius as Origin

In classical geometry, the circle holds a central place, as both an elegant abstraction and a practical reference. Its defining property is the ***radius***: the fixed distance from the center to any point on the perimeter.
From this one measure, all other geometric properties follow.

- The ***circumference*** is derived via $C = 2\pi r$
- the ***area*** via $A = \pi r^2$

Even the identity of the circle itself is inseparable from the presence of a stable, known radius. This radius-first logic is deeply embedded in education, mathematics, and computational geometry. It is conceptually intuitive and mathematically consistent. In a world of compasses, curves, and continuous measurements, the radius serves as both input and anchor.
But this logic assumes a world where ideal shapes can be constructed and measured directly. In digital systems, such as image recognition, medical diagnostics, and computer vision, circles are rarely ideal and radii are almost never directly available. Instead, shapes appear as rasterized forms; fragmented, pixelated, or inferred from edges. The classical radius becomes something to be estimated or reconstructed, introducing error and abstraction into systems that require clarity and speed.

1.2 From Classical to Digital: Why We Need a New Logic

The Geometric Ratio Model (GRM) offers a structural inversion of this traditional approach. Rather than using the radius to define the circle, GRM begins with the bounding frame (typically a square or cube) and derives the radius as a ***consequence of fit***.
In GRM, a perfectly inscribed circle within a square of perimeter 1 SPU (Square Perimeter Unit) has a fixed radius of 0.1250 SPU. This value is not calculated through π, but directly inferred from the relationship between the square's side and the circle's position. It is a structural derivative, not a primary input.
This inversion is more than a mathematical curiosity; it represents a shift in perspective. By prioritizing the frame over the form, GRM aligns geometric interpretation with how digital systems perceive and process shapes.
Instead of asking "*what is the radius?*", the model asks *"how does the shape proportionally fill its space?"* . From this proportional logic, radius-like behavior can be inferred, without estimating curves or measuring invisible points. In this framework, the radius is not lost, but redefined: Not as a number to be inserted, but as a meaningful outcome of spatial structure.

This whitepaper explores that redefinition. We will examine how the radius functions within GRM as a derived quantity, a dimensional operator, and a reciprocal element of geometric interpretation. From proportion to identity, and from pixel structure to design harmony, the radius becomes a window into deeper geometric understanding, anchored not in abstraction, but in measurable form.

2. Limitations of Classical Geometry

Geometry has long been a language of certainty. Since Euclid, we have defined shapes using clear rules and elegant formulas. The circle, in particular, stands as a model of perfection, defined by a center and a constant radius, from which all else follows. Its area and circumference are derived from the well-known constant π, and the radius acts as a stable input across domains.
But this clarity, so useful on paper and in physical design, becomes strained when translated into digital environments.

2.1 Inaccessibility of Primary Measures

In traditional geometry, the radius is assumed to be known. Whether by drawing with a compass or solving a mathematical problem, the radius is the starting point; clean, exact, and central to the identity of the circle.
In digital systems, this assumption no longer holds. When working with raster images, vector graphics, or segmented scans, the radius is rarely given. A shape might look circular to the human eye but provide no access to a true center or measurable radius. Instead, machines must infer this invisible parameter based on the outer form, estimating a core quantity from incomplete or indirect data.
This reversal introduces *fragility*. Instead of working from certainty, digital geometry often begins with uncertainty. A form must be interpreted, its center located, and its dimensions deduced; all without ever knowing if the shape truly fits the classical ideal. What once was a foundation becomes a question mark in digital systems.

2.2 Dependence on Irrational Constants

Classical circle geometry relies on π, an irrational constant with infinite precision. While this works well in theoretical math or controlled environments, it becomes problematic in digital applications.
Digital systems are finite. They rely on discrete units, floating-point approximations, and bit-level representations of number. When calculations involve π, results must be rounded, truncated, or represented with limited precision. This introduces subtle inconsistencies, especially when data is transferred across systems, compared at different resolutions, or scaled dynamically.
What seems mathematically sound in theory becomes numerically unstable in practice.
Even small inconsistencies in π-based calculations can cause noticeable drift in CAD models, graphics rendering, and scientific simulations. The reliance on an irrational constant forces digital systems to simulate rather than truly represent.

2.3 Dimensional Disconnection

In classical geometry, different dimensions are governed by different formulas. A circle's area uses πr^2; a sphere's volume uses $\frac{4}{3}\pi r^3$; and linear dimensions scale separately. There is no internal mechanism that links these expressions proportionally. This dimensional disconnect creates complexity. Scaling a circle to a sphere, or comparing area and volume behaviorally, requires constant switching between formulas and domains. For designers, coders, or AI systems, this increases the cognitive and computational load. The radius does not act as a *dimensional operator*, but merely as a *dimensional placeholder*.
GRM seeks to solve this by embedding radius into a cross-dimensional ratio system, one where length, area, and volume are consistently connected through proportional logic.

2.4 Mismatch with Raster Structures

Perhaps the most critical limitation of classical geometry is that it was never designed for digital representation. A circle on screen is not a curve, it is a pattern of pixels arranged to approximate one. It has no smooth edge, no continuous form, and no true radius in the mathematical sense.
Yet classical geometry continues to interpret these shapes as if they were ideal.
This creates a mismatch between how shapes are represented and how they are analyzed. The circle's identity depends on an invisible radius, while the screen only shows us spatial occupation. What digital systems can truly "see" is not the radius, but how much of the space a shape fills, its proportion relative to a bounding frame. Here, classical logic has no tool to help.

2.5 Toward a Ratio-Based Understanding

These limitations are not failures, they are symptoms of a model designed for a different world.

The geometry we inherited was built for pen and paper, for smooth curves and continuous measurement. In today's world of pixels, voxels, and data grids, we need a system that aligns more closely with what is visible, measurable, and structurally interpretable.

The Geometric Ratio Model (GRM) begins from this recognition. It proposes a new foundation for interpreting shapes, not from radius, but from proportion; not from internal abstraction, but from external structure. The next chapter introduces this logic.

3. The GRM Approach – Bounding Frame First

In classical geometry, shapes are often defined from the inside out. A radius gives rise to a circle; a side defines a square; a height and base yield a triangle. The figure emerges from its parameters, and the surrounding space is considered secondary, context rather than structure.

GRM reverses this orientation. Instead of defining shapes from their internal properties, the Geometric Ratio Model starts with the bounding frame; the outer container in which a shape is inscribed. For two-dimensional shapes, this frame is typically a square; for three-dimensional shapes, a cube. The shape is interpreted in terms of how it fits within this frame; how much space it fills, and how its structure relates to the frame's dimensions.

This shift is not just geometric, it is philosophical. Where classical geometry asks, "*What is the radius of this circle?*", GRM asks, *"How does this shape relate to its surrounding square?"* It is a move from *absolute measurement* to *relative structure*. From internal definition to external relation.

3.1 Deriving the Radius from the Square

At the heart of this approach lies a simple yet profound observation:

If a circle is perfectly inscribed within a square, its radius is exactly half the side of the square.

In GRM terms, this relationship becomes even more structured. Let us define the perimeter of the bounding square as 1 SPU (Square Perimeter Unit). Since the square has four equal sides, each side is 0.25 SPU, and thus the radius of the inscribed circle is: *r = 0.1250 SPU*. This value is not chosen, it is derived. It follows directly from the relationship between the circle and the square. In this model, the radius is no longer an independent quantity but a consequence of spatial configuration.

In GRM, this derived radius becomes a system constant. Whenever a shape is perfectly inscribed, its radius is fixed relative to the frame: *0.1250 SPU* in 2D, and proportionally consistent across dimensions. This turns the radius into a universal unit of proportional behavior, rather than a form-specific variable. This derived radius becomes a cornerstone of proportional reasoning. It allows for consistent interpretation across dimensions, and (unlike the classical approach) requires no knowledge of π.

3.2 Radius Reciprocity in GRM – From Side to Circle, and Back

What makes the GRM radius truly powerful is its *reciprocity*. In classical geometry, the radius leads to the circle. But in GRM, the logic flows in both directions. Knowing the frame, we derive the radius. But equally, by observing a shape's behavior (how much space it fills) we can infer whether it behaves like a circle, and thus what its implied radius must be.

This reciprocal reasoning transforms the radius from a fixed length into a structural fingerprint.

For instance: if a shape fills approximately 78.54% of its bounding square (as a perfect circle would), we can interpret it as functionally circular, even if its edges are irregular or pixelated. This proportion implies a radius of ~0.1250 SPU, not because we measured it, but because we recognize its geometric behavior. This is not estimation. It is pattern-based deduction.

In a digital context, where radius is often invisible, this ability to reason backward from structure to form is revolutionary. It allows systems to interpret shape not by abstraction, but by spatial logic. *The radius becomes not just a line, but a lens.*

3.3 A New Starting Point

By rooting shape identity in the frame rather than the center, GRM establishes a new geometric foundation. This foundation is visible, scalable, and dimensionally consistent. It allows for inference where measurement fails, and interpretation where idealization breaks down. In the next chapter, we explore how this derived radius operates across dimensions,linking perimeter, area, and volume into a coherent and proportional system.

4. Radius as Dimensional Operator

In classical geometry, the radius is a linear measure. It describes distance, nothing more. When used in formulas, it is raised to a power, multiplied by constants, and shaped into new dimensions. But in itself, the radius holds no dimensional agency. It is simply a value inserted into a predefined rule.
In GRM, this changes.
Because the radius is derived from the frame, and because the frame itself is structurally defined across dimensions, the radius becomes *dimensionally expressive*. It connects perimeter, area, and volume through a shared proportional foundation. This makes the radius not just a length, but a *dimensional operator*, a bridge between geometric scales.

4.1 From Line to Surface to Volume

Let us consider a square of 1 SPU perimeter. As established, each side is 0.25 SPU, and the radius of the inscribed circle is 0.1250 SPU. From this, the area of the circle is not calculated using π, but derived via ratio:

Area of inscribed circle = 0.7854 SAU (Square Area Units)

This fixed ratio (π/4) holds regardless of size, as long as the form is truly inscribed. Likewise, in three dimensions, a perfectly inscribed sphere within a cube of 1 SPU^3 has a fixed volume ratio of:

Volume of inscribed sphere = 0.5236 SVU (Square Volume Units)

What links these values is not π alone, but a consistent relationship between the radius and the dimensional context in which it exists.
In GRM, the radius is scaled proportionally in each dimension. The model retains coherence whether we move from 1D to 2D, or from 2D to 3D. *The radius becomes a scaling principle.*

4.2 A Unified Proportional Chain

This dimensional continuity can be summarized as a chain of fixed transformations:

From side → radius:	$r = s / 2$
From side → perimeter (1D):	$P = 4s$
From side → area (2D square):	$A = s^2$
From radius → area (circle):	$A = 0.7854 \times s^2$
From radius → volume (sphere in cube):	$V = 0.5236 \times s^3$

These relationships are not variable, they are structurally locked within the GRM framework. They enable systems to move fluidly between dimensions using proportion as the primary logic. In classical geometry, such continuity requires switching formulas and inserting constants. In GRM, it is embedded in the geometry itself.

Dimensional Note:

In GRM, the radius functions as a dimensional operator, its powers reflect transitions between geometric levels:

- r^1 corresponds to linear structure (SPU),
- r^2 determines 2D proportional coverage (SAU),
- r^3 yields 3D volumetric occupation (SVU).

These dimensional outcomes require no irrational constants and remain consistent across scale, making the radius not only derived, but generative across dimensions.

4.3 Operational Power in Digital Contexts

In digital applications, this dimensional fluency has powerful implications. An AI system identifying a circular form can infer not only its 2D behavior (surface coverage) but its potential 3D interpretation (volumetric fit), using the same proportional base. A visual composition tool can scale elements proportionally across size and depth without recalculating from scratch.
This is not just convenience,it is computational efficiency through structural logic. By treating the radius as a dimensional operator, GRM turns geometry into a scalable language; one that machines can read, extend, and apply without needing the full weight of classical abstraction.

4.4 Ratio Enables Comparison

One of the most powerful consequences of defining the radius proportionally is that it enables structural comparison across shapes.

- In classical geometry, comparing different shapes, say, a circle and a hexagon, requires transforming them into shared units or using approximations. Their differences are expressed numerically, but not structurally.
- In GRM, shapes can be compared directly based on how much of the bounding frame they occupy.
    - A circle inscribed in a square fills exactly 0.7854 SAU.
    - A regular hexagon inscribed in the same square fills 0.8660 SAU.

  These values are not approximations, they are identity ratios.
  As a result, two shapes that look similar but have different ratios can be interpreted not just as different in size, but different in nature. Ratio reveals structure. Ratio makes deviation meaningful.

This comparative logic also allows digital systems to recognize shapes by relative behavior, rather than absolute dimension. Whether an object is large or small, if it fills approximately 78.5% of its bounding square, it behaves like a circle; regardless of pixel resolution or scale.
Thus, the radius, derived from the bounding frame, becomes a bridge between dimensional logic and structural comparison.

In the next chapter, we explore how this ratio-based understanding of radius enables new forms of interpretation—shapes not just measured, but recognized by their structural fit.

5. Ratio-Based Interpretation and Identity

In classical systems, shape recognition often depends on labels, templates, or predefined mathematical forms. A circle is defined by its radius. A hexagon by its side length. Irregular shapes are categorized by comparing them to idealized models, using techniques like curve fitting or feature extraction.
But in practice, especially in digital environments, shapes are rarely perfect.
A scanned medical image might show a near-circular organ. A design element might almost, but not quite, form a hexagon. In these cases, classical geometry offers no native way to describe structural closeness. A form either fits the formula, or it doesn't.
The Geometric Ratio Model introduces a new form of reasoning: *Identity by ratio*. Not by name, not by formula, but by structural behavior relative to a bounding frame.

5.1 Fuzzy Classification and Radius Deviation

In GRM, the ratio between a shape's area and the area of its bounding square (SAU) becomes its *geometric fingerprint.*

- A perfect circle has a ratio of 0.7854 SAU.
- A regular hexagon fits at 0.8660 SAU.
- A square fills 1.0000 SAU.

Now consider a shape that fills 0.7700 SAU.
Is it a circle? Not exactly.
Is it a square? Certainly not.
But it is close enough to the ideal circle ratio to suggest a circular identity.

This is what GRM calls fuzzy classification:
Shapes are interpreted not by rigid matching, but by proximity to known ratio signatures.
A deviation from 0.7854 does not imply error. It implies structure, variation, deformation, transformation.
This interpretation gives systems a more nuanced vocabulary. A shape is not just "circle or not," but "circle-like within 2%." This approach transforms geometric deviation from a source of failure into a source of information.

5.2 The Natural Fit Ratio (NFR) as Interpretative Metric

The Natural Fit Ratio (NFR) builds on this logic. Where GRM ratio describes how much space a shape occupies in a square, NFR interprets how well that occupancy aligns with known structural norms.

- If a shape fills 78.5% of its square, it exhibits natural circular fit.
- If it fills 86.6%, it aligns with a hexagonal structure.
- Values in between reveal hybrid forms, approximate fits, or shape blends.

NFR gives geometry a semantic layer. It allows digital systems, and humans, to understand form by how it behaves, not just what it's called. In visual design, this leads to harmony. In AI, it improves recognition. In diagnostics, it sharpens differentiation.
The radius remains implicit in these evaluations. It is not measured, but emerges from the ratio. The form speaks its identity through proportion.

5.3 Radius-Based Recognition in Pixel and Visual Systems

In raster environments, where everything is built from pixels, classical measures are often inaccessible. No radius can be measured directly. No perfect curve exists.
But GRM does not need a curve. It only needs the number of pixels inside the shape, and the number of pixels in its bounding square. The ratio between them is enough. From that, the system can infer:

- If the shape behaves like a circle (≈ 0.7854),
- If it resembles a square (≈ 1.0000),
- Or if it lies somewhere in between.

This allows for classification without edge tracing, curve fitting, or reconstruction. It is especially powerful in machine learning contexts, where input data is noisy, incomplete, or low resolution. Instead of abstract modeling, the system can reason from structural fit. By rooting identity in proportion, GRM makes pixel data meaningful. *The radius becomes not a line to measure, but a logic to infer.*

In the next chapter, we explore how this ratio-based logic fits into the broader framework of digital geometry, revealing how the radius, once a local measure, becomes a structural operator in the logic of machines.

6. Radius Within the Digital Geometry Paradigm

The history of geometry is a history of abstraction. From Euclid's definitions to modern algebraic models, geometry has often aimed to describe the ideal, to capture perfection through clean forms, continuous curves, and symbolic notation. This legacy shapes how we think, teach, and calculate. But it also creates a blind spot.
Most of what we now see, interpret, and model happens in digital systems. And in these systems, abstraction becomes distortion. The forms we encounter (scanned organs, satellite images, design elements, 3D renders)are not perfect shapes. They are discretized structures, made of pixels, points, or

polygons. They cannot express infinite precision. They operate within a grid, and what matters is not absolute measure, but relative fit. In this world, classical geometry begins to crack.

6.1 A Shift in Geometric Thinking

The Geometric Ratio Model (GRM) does not seek to fix classical geometry. It seeks to reorient it, to shift the starting point from internal abstraction to external structure. And nowhere is this clearer than in the treatment of the radius.

- In classical logic, the radius is assumed. It is the origin of all subsequent interpretation.
- In GRM, the radius is derived; not from measurement, but from structure.
  Its value is inferred from the frame and the fit of the shape. This is not just a change in calculation. It is a change in paradigm.

- Where classical geometry says: *"Here is a radius; now calculate the area."*
- GRM says: *"Here is a shape that occupies 78.54% of its square; therefore, its behavior is circular, and its implied radius is 0.125 SPU."*

This reasoning is not theoretical, it is operational. It is designed for systems that work with partial data, visual fragments, and pixel grids. It transforms the radius from an invisible line into a visible consequence.

6.2 Geometry for Machines, Not Just Mathematicians

What makes this approach powerful is that it aligns with how machines see. AI systems do not measure radii. They do not perceive perfect centers or infinite curves. What they "see" is how much of a grid is filled. How a shape fits. Whether its ratio matches known behaviors. And that is exactly what GRM quantifies.

In this light, the radius becomes an interface between classical and digital reasoning. It preserves the structure of traditional geometry (circle, square, frame) but expresses it in a way that machines can directly understand and use.

This is the essence of the digital geometry paradigm:

- From lines to logic.
- From measurement to meaning.
- From assumptions to derivations.

In this new paradigm, the radius survives not as a starting point, but as an outcome.
Not given, but earned; through proportion, through fit, and through structural coherence.
In this light, GRM aligns not only with visual recognition, but also with broader digital geometry frameworks.Its ratio-based reasoning is structurally compatible with systems used in computer-aided design (CAD), medical image processing, and AI-driven interpretation pipelines, domains where relational geometry and spatial efficiency are paramount.
Rather than simulating classical abstraction, GRM offers a native logic for digital form.

The next chapter explores how this logic translates into real-world systems. We move from paradigm to practice: How does this reimagined radius help machines see? How does it improve recognition, efficiency, and structure? What becomes possible when proportion, not abstraction, leads the way?

7. Practical Implications and System Applications

While the Geometric Ratio Model (GRM) introduces a new way of reasoning about form, its value lies not only in conceptual clarity, but in operational power. By reimagining the radius as a derived and ratio-based element, GRM opens the door to a broad range of practical applications across domains where geometry, shape recognition, and visual logic intersect.

This chapter highlights how GRM's radius logic translates into measurable advantages in real-world systems.

7.1 Pixel-Based Shape Recognition Without Reconstruction

In many digital systems (e.g. image recognition, computer vision, segmentation tools) shape classification relies on complex pipelines: edge detection, curve fitting, contour tracing, center estimation. These methods attempt to reconstruct classical parameters (like radius) from noisy, raster-based input.
GRM removes the need for reconstruction.
By comparing how much of the bounding square a shape fills, systems can classify forms directly. A fill ratio near 0.7854 suggests a circular structure. No center, no radius, no curvature required. This makes GRM particularly suited for low-resolution imagery, noisy data, and real-time applications where speed matters more than precision.
The radius is not measured. It is inferred from proportion; a lighter, faster, and more robust alternative.

7.2 Ratio-Based Deviation Mapping in Diagnostics

In fields like medical imaging, slight deviations from geometric ideals can signal structural anomalies. But traditional geometry lacks tools to describe "almost circular" or "subtly deformed" in meaningful terms. GRM provides a language for these subtleties.
If a scanned region occupies 0.76 SAU, the system can report a 2% deviation from ideal circularity. This is not noise, it is directional information. It tells radiologists or diagnostic algorithms how, and by how much, a shape diverges from normal.
GRM thus enhances anomaly detection, not by adding complexity, but by enabling comparison through ratio logic.

7.3 AI-Driven Design and Aesthetic Guidance

Design tools increasingly use AI to generate visual compositions, user interfaces, or product shapes. But how do these systems understand "visual balance" or "natural fit"?
GRM offers an interpretable metric. Using ratios like NFR (Natural Fit Ratio), systems can suggest adjustments that improve visual harmony; aligning curves, correcting distortions, or matching user intent.
A radius inferred from fit (rather than imposed) reflects **organic structure**, making outputs feel more "human" despite being machine-generated.
This gives AI a structural intuition grounded in form, not in formula.

7.4 Efficiency in Machine Reasoning

In embedded systems, real-time robotics, and AR/VR environments, computational resources are limited. Floating-point operations involving π, curve-fitting, or inverse trigonometry are costly.
GRM's fixed ratios, like 0.1250 SPU for radius, 0.7854 SAU for area, can be pre-indexed and directly compared.
This yields:

- Lower computational load
- Faster execution time
- Greater robustness across resolutions

Geometry becomes not just precise, but performant.

7.5 Structural Consistency Across Dimensions

When designing forms that transition across dimensions (e.g. such as 2D-to-3D modeling, unfolding surfaces, or simulating growth) GRM provides a single consistent system. The radius derived in 2D leads naturally to volumetric interpretation in 3D, using SVU (e.g., 0.5236). Systems that must model or predict dimensional behavior can benefit from this structural continuity; reducing the need for domain-specific adaptations.

These applications reveal the core strength of GRM: *It offers a logic of shape that is both mathematically rigorous and systemically efficient.*
By repositioning the radius from given to derived, from absolute to relational, GRM turns geometric identity into something digital systems can see, understand, and apply,natively.

In the next and final chapter, we return to the conceptual core, reframing the radius as not only a quantity, but a principle for understanding.

8. Conclusion – Radius Reimagined

For centuries, the radius has served as a trusted starting point, a simple line that defined the identity of the circle, linked dimensions, and anchored formulas. In doing so, it symbolized the geometric ideal: *easurable, precise, and universal.*
But in a digital world, precision looks different. Shapes are no longer theoretical. They are fragmented, pixelated, and context-dependent. Measurement is no longer absolute, but inferred. And interpretation demands more than formulas; it requires structure, context, and proportion.
This is where the Geometric Ratio Model (GRM) offers a shift. By redefining the radius as a derived quantity, GRM positions it not as a fixed input, but as a consequence of structural relation. A shape that fills 78.54% of its bounding square does not just resemble a circle; it behaves like one. Its radius is not measured; it is *recognized*. Its identity is not assigned; it is *revealed* through ratio.
This reframing unlocks new forms of reasoning:

- ***Dimensional coherence***, where radius links 1D, 2D, and 3D seamlessly.
- ***Fuzzy classification***, where near-forms are interpreted by proximity to ideal ratios.
- ***Pixel-based recognition***, where structure replaces abstraction.
- ***Machine-native geometry***, where visual systems no longer simulate understanding, but embody it.

In this logic, the radius becomes more than a length. It becomes a lens; a way to interpret shape, identity, and meaning through proportion.
GRM does not eliminate the radius. It elevates it, from a mathematical given to a geometric consequence. From a tool for calculation to a framework for comprehension. This redefined role of the radius also opens new avenues for future exploration, such as the structural behavior of non-inscribed forms, irregular deviations, and composite shapes where proportional identity is less obvious but no less significant. In doing so, it invites us to look again at the most familiar of forms, and to see not just lines and curves, but ratios, relations, and a new kind of geometric truth.

Appendix A – Derivations and Ratio Tables

The Geometric Ratio Model (GRM) derives its consistency from a system of fixed ratios. These ratios express how standard geometric shapes behave when perfectly inscribed within a square (2D) or cube (3D), and form the foundation for proportional reasoning across dimensions. All derivations assume the bounding frame (square or cube) as the reference unit.
The frame defines:

- **1 SPU** (Square Perimeter Unit) in 1D,
- **1 SAU** (Square Area Unit) in 2D,
- **1 SVU** (Square Volume Unit) in 3D.

A.1 Radius as a Derived Quantity

Shape	Frame Type	Derived Radius	Remark
Circle in square	Square	0.1250 SPU	Radius = half of side = $\frac{0.25}{2}$
Sphere in cube	Cube	0.1250 SPU	Same proportional logic as 2D

In both 2D and 3D, the derived radius is structurally fixed and scale-independent within GRM.

A.2 Area and Volume Ratios of Inscribed Shapes

Shape	Dimension	Ratio (relative to frame)	Unit	Formula
Circle	2D	0.7854	SAU	π / 4
Square	2D	1.0000	SAU	$side^2$ / $side^2$
Regular Hexagon	2D	0.8660	SAU	3√3 / 4 ≈ 0.8660
Sphere	3D	0.5236	SVU	π / 6
Cube	3D	1.0000	SVU	$side^3$ / $side^3$

A.3 Dimensional Scaling Through r

In GRM, the radius scales meaningfully across dimensions:

Power of r	Interpretation	Derived Value (from 0.125 SPU)
r^1	Linear radius (1D logic)	0.1250
r^2	Proportional surface coverage (2D)	0.0156
r^3	Volumetric occupation (3D)	0.00195

These values correspond to behavior **within a frame of 1 SPU, SAU or SVU**, and can be scaled proportionally.

A.4 Structural Constants in GRM

Constant	Value	Meaning
π/4	0.7854	Area ratio of circle in square (SAU)
π/6	0.5236	Volume ratio of sphere in cube (SVU)
3√3 / 4	0.8660	Area ratio of regular hexagon in square
r_SP	0.1250	Radius as fraction of square perimeter

These constants allow GRM-based systems to compute, classify, and compare shapes structurally—without relying on floating-point estimation or irrational starting values.

Appendix B – Visual Reference and Structural Diagrams

The Geometric Ratio Model (GRM) is fundamentally a visual and spatial model, designed to operate not only through symbolic formulas but also through interpretable structures. This appendix is intended to provide reference illustrations that show how key GRM ratios emerge from geometric relationships within a square or cube.

At this time, the precise generation of GRM-consistent visuals (e.g. perfectly inscribed shapes, pixel-based deviations, or dimensionally scaled diagrams) is technically limited in available image-generation tools. Therefore, the visual figures for this appendix are currently under development.

They will include:

- **Perfectly inscribed shapes** with exact fill ratios (e.g. 0.7854 SAU for circles),
- **Ratio deviation maps** for fuzzy classification,
- **1D–2D–3D scaling layers** based on radius progression,
- **Pixel-frame overlays** illustrating recognition without reconstruction.

These visual materials will be released in a future revision of this paper and made available through the GRM knowledge platform at www.inratios.com.

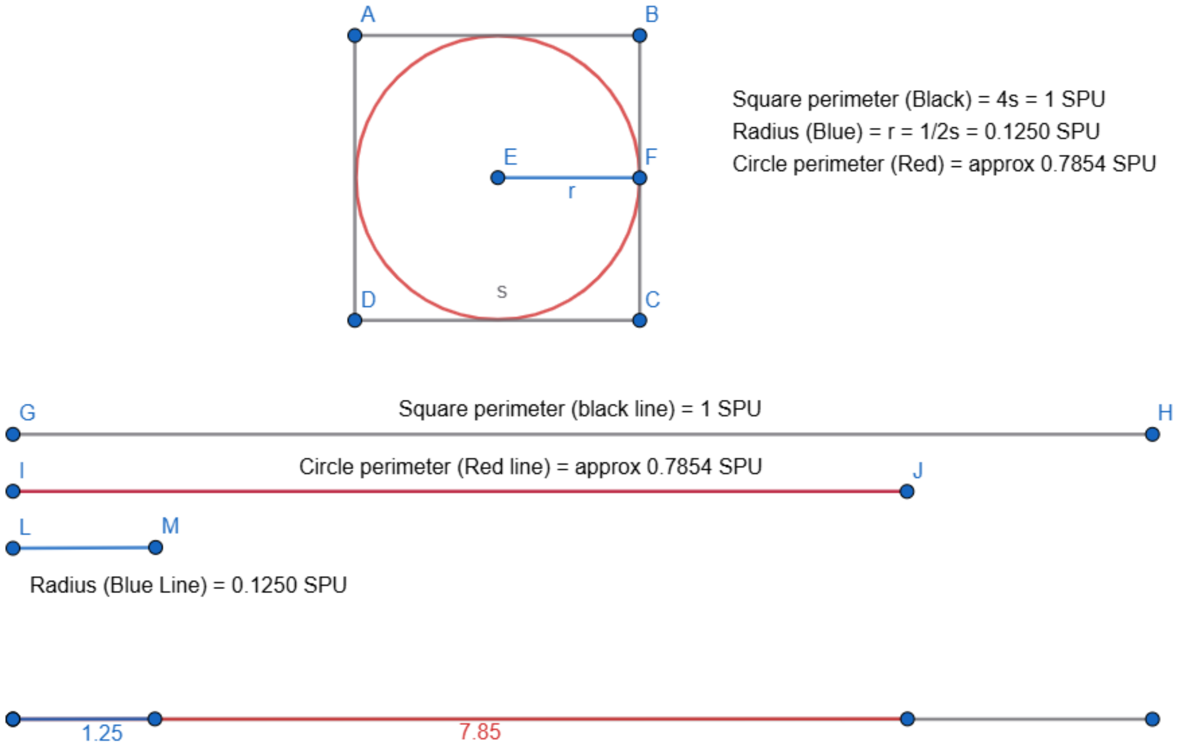

Figure B.1 – A circle perfectly inscribed in a square.
The square has a total perimeter of 1 SPU, making each side 0.25 SPU. The radius is derived directly: r = s / 2 = 0.1250 SPU. The circle occupies approximately 0.7854 (π / 4) of the square's area and perimeter. The lower bars compare the full square perimeter (black), circle perimeter (red), and radius (blue). This figure illustrates GRM's principle of deriving measurable relationships from visible boundaries—no irrational constants required.

