WHITE PAPER

The Geometric Ratio Model (GRM)

A New Metric for Proportion, Shape, and Dimension

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Summary

This document introduces the GRM model as a scale-free, visually robust, and dimensionally consistent approach to geometry.

The model replaces abstract irrational constants, such as π, with fixed ratios within standard shapes (square and cube) and offers applications in education, engineering, visualization, design, and systems thinking.

It is both didactically accessible and theoretically scalable to higher dimensions.

Version update notes – From 2.0 to 3.0

This third version of the GRM whitepaper represents a significant evolution from version 2.0, both in clarity and in practical applicability. The core logic of the Geometric Ratio Model remains unchanged, but several important refinements have been implemented:

- **Improved structure and tone:** The writing style has been updated for consistency, clarity, and narrative cohesion, aligning with academic and technical standards while remaining accessible.
- **Expanded dimensional framework:** The foundational section has been restructured to better explain the 1D–2D–3D transition, with new visuals illustrating SPU, SAU, and SVU as coherent dimensional units.
- **New application domains and visuals:** Four key domains, Design & CAD, AI & Classification, Education, and Measurement, have been expanded with real-world examples and custom illustrations demonstrating the added value of GRM logic.
- **Rewritten appendices:** Appendices A and C have been fully rewritten to match the new tone and to better explain the geometric identity of the hexagon and classification tolerances using GRM ratios.
- **Updated tables and summaries:** Table 4.5 and accompanying summaries have been reformulated to better align with the discussed content, focusing only on domains elaborated in this version.
- **Visual consistency and corrected figures:** All illustrations have been recreated using GRM principles, ensuring correctness in ratio application, especially in dimensional enclosures (e.g., circles and spheres perfectly inscribed within containers).

These updates collectively elevate the whitepaper to a mature reference document, suitable for publication, academic discussion, and applied implementation in digital systems.

Peer review invitation

This whitepaper represents version 3.0 of the Geometric Ratio Model (GRM), a framework developed to enable fixed-ratio reasoning in geometry, with specific relevance to digital systems, education, and design logic.

The content has been reviewed internally for consistency, correctness, and practical alignment with real-world applications. However, as an evolving concept that bridges classical geometry and proportional logic, the GRM remains open to critical feedback, empirical validation, and collaborative refinement.

We invite experts, researchers, and practitioners from the following fields to review this work and provide constructive feedback:

- ***Geometry and mathematical logic***
- ***Computer-aided design (CAD) and engineering***
- ***Artificial intelligence and pattern recognition***
- ***Educational methodology and visual reasoning***
- ***Philosophy of mathematics and systems thinking***

If you are interested in contributing to the academic or technical review of this model, or wish to collaborate on implementation, experimentation, or publication, please contact:

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Your feedback will directly support the future development, validation, and responsible deployment of GRM logic in both theoretical and applied contexts.

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1. Introduction and core vision

1.1 Background

For centuries, the mathematical constant π (pi) has been central to geometry. Defined as the ratio of a circle's circumference to its diameter, π is an irrational number that cannot be expressed exactly in decimal form. While π is mathematically robust, its application in education, digital systems, and geometric reasoning often presents challenges: it is abstract, indirectly measurable, and inherently resistant to intuitive interpretation.

In contemporary context, where digital simulation, visual reasoning, and hands-on education increasingly dominate, the need for alternative geometric approaches has grown. Geometry is no longer confined to theoretical derivation; it must also be actionable, scalable, and perceptually grounded. The Geometric Ratio Model (GRM) arises in response to this shift: not to redefine mathematics, but to reformulate geometry in a way that better serves modern cognitive and computational frameworks.

1.2 Purpose and motivation

The purpose of the GRM model is to reframe the way geometric properties (such as perimeter, area, and volume) are represented and applied. Rather than expressing these through abstract constants, the model proposes a system of fixed ratios within simple, universally measurable forms: the square (2D) and the cube (3D). These shapes serve as standard reference units across dimensions:

- The square perimeter becomes 1 SPU (Square Perimeter Unit)
- The square area becomes 1 SAU (Square Area Unit)
- The cube volume becomes 1 SVU (Square Volume Unit)

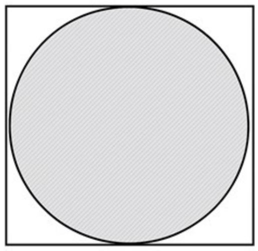

Figure 1.1 – *A* circle inscribed in a square occupies 0,7854 of its area.

Within this framework, other shapes (such as the circle or sphere) are expressed in relation to the square or cube that encloses them. For instance, a circle inscribed within a square has a fixed perimeter ratio of $\pi/4 \approx 0.7854$ *SPU* and an area ratio of $\pi/4 \approx 0.7854$ *SAU*. A sphere within a cube yields a volume ratio of $\pi/6 \approx 0.5236$ SVU.

The GRM does not replace π as a constant. Instead, it recasts π-derived results into relative metrics that can be observed, measured, and applied across disciplines. This makes the model both didactically intuitive and computationally adaptable.

1.3 Core vision

The core vision of the GRM model is to establish a scale-free, visually grounded, and dimensionally consistent geometry. It emphasizes relational properties over absolute values, and structural containment over symbolic derivation. The model introduces the following three guiding principles:

- ***Visual Intuition***: Shapes are identified and compared through their relationship to enclosing forms, not through abstract computation.
- ***Relational Logic***: All measurements are expressed as fixed ratios within a standard container, offering universality and comparability.
- ***Dimensional Scalability***: The model is inherently extensible across 1D, 2D, 3D, and beyond, using a single proportional language.

By rooting these principles in simple geometric structures, the GRM model creates a bridge between theoretical rigor and applied insight. It supports geometric reasoning that is both precise and perceptually accessible.

It is important to emphasize that the GRM model is not intended to redefine classical mathematics. Rather, it provides a complementary metric framework that better serves digital computation, educational practice, and systems-based thinking. The model retains compatibility with traditional geometric theory while enabling novel pathways for learning, simulation, and design.

1.4 Outlook on higher dimensions

Although the GRM model focuses primarily on the first three spatial dimensions, its logic is naturally extendable. The ratio between an inscribed hypersphere and its enclosing hypercube, such as in four-dimensional space, can be derived and expressed as a fixed metric, for example:

$$VE_4 = \frac{\pi^2}{32} \approx 0.3084$$

This extension demonstrates that the GRM model can evolve into a universal geometric framework, applicable across dimensions and domains. It opens new avenues for research in multidimensional design, digital modeling, and even mathematical philosophy.

The following chapters will elaborate the GRM model step by step, examining its foundations, comparing it with classical formulations, exploring its applications, and reflecting on its broader significance.

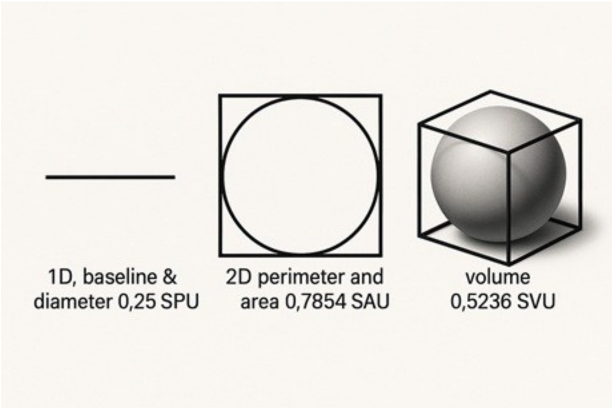

Figure 1.2 GRM ratios

2. Foundations of the GRM Model

2.1 The standard form: The square and the cube

The Geometric Ratio Model (GRM) begins with a simple yet powerful premise: that all geometric quantities, length, area, and volume, can be referenced to a single, well-defined standard shape. In two dimensions, this is the square; in three dimensions, the cube. These shapes form the metric backbone of the GRM system.

By assigning fixed measurement units to these standard forms, we establish a universal framework that is both dimensionally consistent and intuitively visual:

- The perimeter of a square is defined as ***1 SPU*** (Square Perimeter Unit)
- The area of the square is defined as ***1 SAU*** (Square Area Unit)
- The volume of the cube is defined as ***1 SVU*** (Square Volume Unit)

These values are not arbitrary. They represent the full metric content of a shape when its side length

$S = 1$. Within this standardized configuration, other shapes (such as circles, spheres, or polygons) can be inscribed, compared, and interpreted using fixed ratios.

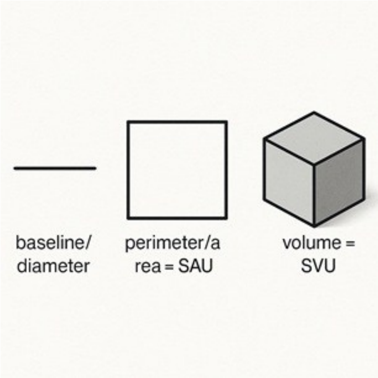

Figure 2.2.1 *Square and cube, each annotated with perimeter = 1 SPU, area = 1 SAU, volume = 1 SVU*

This normalization process allows the GRM to eliminate dependency on absolute units or irrational constants, and instead reason proportionally: *"How much of the container does this shape occupy?"*

For example, *when a circle is perfectly inscribed in a square of side length 1, its perimeter becomes approximately 0.7854 SPU, and its area 0.7854 SAU. A sphere, when inscribed in a cube, occupies 0.5236 SVU.*

These fixed proportions form the foundation of GRM's ratio logic. The model no longer depends on π as a symbolic constant; instead, it expresses geometric identity as a dimensionless relationship between a shape and its enclosing structure.

This proportional reasoning extends seamlessly across dimensions, as the next sections will demonstrate.

Table1.1 *- Standard Ratios in the GRM Model*

Dimension	Measurement Aspect	Standard Shape	Formula (s=1)	Ratio	Unit
1D	Length (perimeter)	Square	$4s$	1	1 SPU
2D	Area	Square	s^2	1	1 SAU
3D	Volume	Cube	s^3	1	1 SVU

Supplementary overview: Ratios in standard and inscribed shapes

The table above shows fixed ratios between standard geometric shapes (the square and cube) and their perfectly inscribed counterparts (such as the circle and sphere), assuming a side length of $s = 1$. These ratios are normalized to the enclosing square or cube and represent **dimensionless constants** within the GRM system. This provides insight into how SPU, SAU, and SVU function as universal, scalable reference units for geometric identity and measurement.

2.2 Enclosed shapes as ratio carriers

One of the key innovations in the Geometric Ratio Model (GRM) lies in its treatment of shapes not as isolated entities, but as ratios within a defined boundary. Specifically, shapes that are *perfectly inscribed* within a square (2D) or cube (3D) take on a fixed ratio relative to that container. This transforms them from absolute figures into dimensionless carriers of geometric identity.

The most familiar example is the **circle**, inscribed within a square of side length $s = 1$. Its perimeter and area are no longer calculated using π, but are instead expressed directly as:

- **Perimeter**:

$$Ratio = \frac{\frac{\pi s}{2}}{4s} = \frac{\pi}{4} \approx 0.7854\, SPU$$

- **Area**:

$$atio = \frac{\frac{\pi s^2}{4}}{s^2} = \frac{\pi}{4} \approx 0.7854\, SAU$$

This fixed value (*0.7854*) serves as a universal identity for the circle, grounded in its relationship to the square. It reflects not the raw dimensions of the shape, but how it behaves *within a standardized reference unit*.

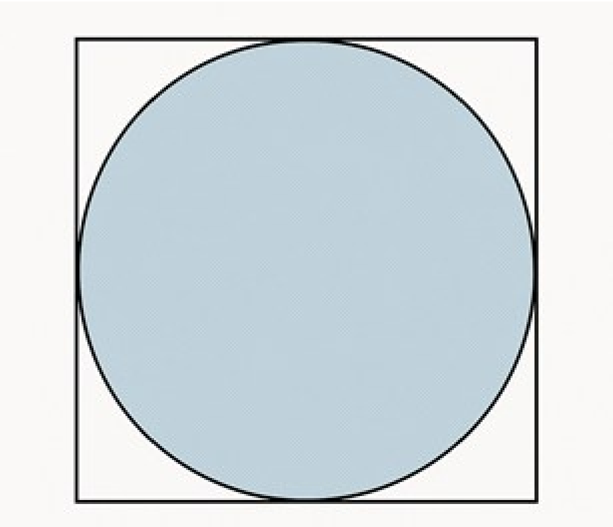

Figure 3.2.1 - A circle inscribed in a square occupies 0.7854 of its area

Additional enclosed forms

The logic of proportional identity can be extended to other shapes:

- A regular hexagon inscribed in a square occupies approximately 0.8660 SAU.
- A regular triangle (equilateral) inscribed in a square covers exactly 0.5000 SAU.
- A sphere inscribed in a cube occupies 0.5236 SVU.

These values are invariant: they hold true regardless of the actual size of the container, as long as the shape remains perfectly inscribed. This makes GRM particularly powerful in digital, scalable, and comparative application; such as computer vision, design validation, and geometry-based AI.

***Tabel 2.2** – Fixed Ratios of Enclosed Forms*

Dimension	Shape	Ratio	Unit
2D	Circle (inscribed)	0.7854	SAU
2D	Hexagon (inscribed)	0.8660	SAU
2D	Triangle (inscribed)	0.5000	SAU
3D	Sphere (inscribed)	0.5236	SVU

These values serve as geometric signatures. Within the GRM model, they enable fast, dimensionless recognition, classification, and comparison, without reliance on irrational constants or complex recalculation. Each ratio represents not a formula, but a fixed identity, valid as long as the enclosure is perfect.

In later sections, we will explore how this logic applies in practical fields, from AI-assisted shape detection to scalable design evaluation.

2.3 Dimensional consistency

A defining strength of the Geometric Ratio Model (GRM) is its internal coherence across dimensions. Whether measuring a line, a surface, or a volume, the GRM maintains a unified metric logic. This dimensional consistency is achieved by expressing each geometric property (perimeter, area, or volume) as a fixed ratio within a standard container: the square (2D) or cube (3D).

The GRM system begins in 1D with the **Square Perimeter Unit (SPU)**, defined by a square with side length $s = 1$, and a perimeter of $s = 4$. By normalizing this perimeter to 1 SPU, any enclosed or related 1D figure can be compared proportionally. In 2D, the area of the square becomes 1 SAU; in 3D, the cube's volume becomes 1 SVU.

This layered structure allows the same proportional logic to be applied seamlessly as complexity increases:

- ***1D (SPU):*** A line segment occupies a share of the square's perimeter.
- ***2D (SAU):*** A shape such as a circle, triangle, or hexagon occupies a share of the square's area.
- ***3D (SVU):*** A volumetric shape such as a sphere occupies a share of the cube's volume.

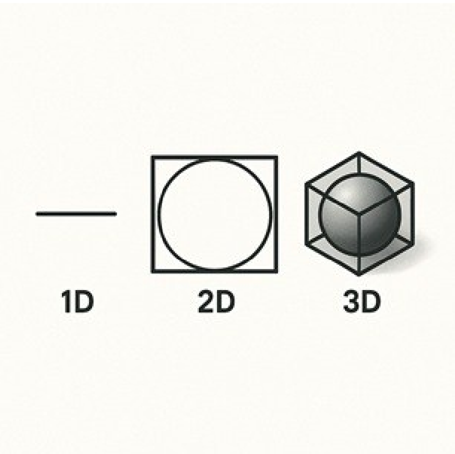

***Figure 2.4** – GRM logic across 1D, 2D and 3D*

This diagram illustrates the GRM logic across 1D, 2D, and 3D.
Each dimension has a corresponding fixed ratio, which defines how much of the standard container is occupied by the enclosed form.

Table 2.3 *– GRM logic*

Dimension	Shape Example	Ratio	Unit
1D	Line	0.2500	SPU
2D	Circle	0.7854	SAU
3D	Sphere	0.5236	SVU

These ratios are scale-free and dimensionless. They allow for meaningful comparison and classification across dimensional levels, without reference to radius, diameter, or irrational constants. Instead, identity and measurement are derived from the proportional occupation of a standardized space.

This consistency forms the conceptual spine of GRM: from simple lines to complex volumes, everything is expressed through the same language of bounded proportion.

In later sections, this coherence will enable GRM to be applied in domains such as AI classification, 3D modeling, and geometry education, where transitions between dimensional levels must be intuitive and computationally efficient.

2.4 Measurability and visualizability

A key advantage of the Geometric Ratio Model (GRM) lies in its direct measurability and visual coherence. Unlike classical formulas that rely on internal parameters such as radius, height, or π-based calculations, the GRM expresses all geometric identities as ratios within visible structures.

These ratios are not abstract constructs. They can be physically measured, approximated, and observed using simple tools:

- In 1D, perimeter proportions can be measured with a ruler or string, comparing segments to a square's total perimeter (SPU).
- In 2D, areas can be assessed using grid tiles, pixel counts, or transparent overlays, matching the area of a circle or triangle against the square (SAU).
- In 3D, volume can be evaluated via water displacement, voxel counts in imaging, or CAD-integrated bounding boxes, comparing the filled volume to that of the cube (SVU).

This visual logic is particularly powerful in educational and digital environments. Students, designers, and AI systems alike can detect and validate shapes based on visible containment and structural fit, rather than symbolic inference.

Table 2.4 *- Example applications*

Context	Method	Observable Unit
Education	Cut-out shapes in cardboard or transparent overlays	SAU
CAD & Design	Bounding boxes and auto-fit routines	SPU / SAU / SVU

Context	Method	Observable Unit
AI & Image Processing	Pixel/voxel ratios, segmentation masks	SVU, tolerance bands
Medical Imaging	Voxel segmentation and bounding cube analysis	SVU
Physical Prototyping	Liquid displacement (Archimedean) comparison	SVU

Unlike π, which cannot be physically isolated or measured directly, GRM ratios such as 0.7854 (circle) or 0.5236 (sphere) can be experimentally confirmed using basic geometry and observable reference shapes. This makes the model inherently verifiable, even in low-tech or didactic contexts.

It also bridges the gap between theoretical and applied geometry, supporting workflows where visual alignment and relational accuracy are more critical than symbolic perfection.

In short: the GRM does not merely define shapes, it reveals them.

2.5 Precision and error margins

The Geometric Ratio Model (GRM) is built upon fixed, rational ratios, such as 0.7854 ($\pi/4$) for a circle or 0.5236 ($\pi/6$) for a sphere, defined relative to their enclosing square or cube. These values, though derived from classical constants, are expressed in a dimensionless and scale-independent manner within the GRM system.

While these ratios are technically approximations, their precision is more than sufficient for most applications in education, design, computation, and classification.

Table 2.5 - *Practical Comparison with Classical π-based Methods*

Use Case	Classical Result (π-based)	GRM Ratio	Absolute Error (4 decimals)
Area of inscribed circle (s = 1)	$\pi/4 = 0.7854$	0.7854	0.0000
Volume of inscribed sphere (s = 1)	$\pi/6 = 0.5236$	0.5236	0.0000
Circumference of circle (s = 1)	$\pi = 3.1416$	$4 \times 0.7854 = 3.1416$	0.0000

In practical terms, the difference is negligible. The GRM expresses these relationships as fixed multipliers against a defined unit, avoiding repeated recalculation and reducing dependency on irrational constants.

Why this matters

In digital systems and educational contexts, floating-point approximations of π and complex internal parameters (radius, height, etc.) often introduce:

- rounding errors,
- inconsistent scaling,
- and unnecessary computational load.

By contrast, GRM uses predefined, validated ratios with direct application across:

- design validation (e.g., does this object fill ~78% of its square?),
- AI classification (e.g., confidence scoring based on match with known GRM ratios),
- toleranced reasoning (e.g., deviation within ±0.03 of 0.5236 is likely spherical),
- and educational models where clarity and reproducibility are crucial.

Looking ahead: Toward ratio-based tolerances

While GRM begins with ideal, fixed ratios, future sections will introduce tolerance bands around each canonical value. These will allow:

- graded classification of imperfect or noisy shapes (e.g., in imaging),
- confidence scoring based on proximity to ideal ratios,
- and practical handling of deviation in manufacturing, detection, and modeling.

By doing so, the model remains both mathematically grounded and operationally robust, especially when applied in real-world or digital scenarios where precision has contextual boundaries.

2.6 Practical benefits of the GRM Model

Beyond its theoretical clarity, the Geometric Ratio Model offers tangible advantages in practical, educational, and digital settings. By reframing geometry through fixed, relational ratios, the GRM unlocks a more usable, verifiable, and scalable approach to shape and dimension.

What does this offer?

1. ***Faster and clearer calculations***
   With fixed ratios replacing complex π-based formulas, only a single multiplication factor is needed. This saves time and reduces both human and computational errors, ideal for fast evaluation, programming, and teaching.
2. ***Simplification in education and training***
   Students and technical learners work with concrete values (e.g. 0.7854 or 0.5236) rather than abstract constants. This makes geometry more visual, measurable, and conceptually accessible, especially in early education, vocational training, or cross-disciplinary learning.
3. ***Consistency in digital systems***
   Fixed ratio values (e.g. 0.7854 SAU or 0.5236 SVU) can be directly implemented in digital environments like CAD software, AI models, or geometry processors. They reduce floating-point instability and ensure scalable, predictable logic across dimensions.
4. ***Real-world measurability and validation***
   The GRM's ratios can be verified through tangible methods: a ruler for perimeter, tiles for surface, or water displacement for volume. This bridges mathematical theory and physical observation, ideal for prototyping, demonstration, and classroom experiments.

These benefits show that GRM is not just a new way to think about geometry, but a practical system ready to be taught, implemented, and measured; across domains and dimensions.

3. Comparison with classical geometry

3.1 Complementary logic, not replacement

The Geometric Ratio Model (GRM) does not seek to replace classical geometry, nor to challenge the foundational truths of Euclidean mathematics. Instead, it provides a complementary lens, a proportional framework that simplifies application, comparison, and classification in domains where abstraction becomes a barrier.

Traditional geometry relies heavily on irrational constants, symbolic formulas, and internal parameters like radius, height, and angle. These are theoretically exact, but often inaccessible in real-world or digital contexts. GRM translates these classical properties into bounded ratios that can be observed, measured, and applied directly.

Classical formulas remain valid; the GRM simply reframes them for a different paradigm of use.

3.2 Why digital systems require a different lens

In digital environments, geometry is not symbolic; it is rendered, visualized, segmented, compared, and scaled. These processes benefit not from mathematical exactness per se, but from operational consistency and visual clarity.

Examples include:

- CAD tools that operate on bounding boxes and vector scaling
- AI vision systems that detect shape classes via pixel ratios
- Educational platforms that teach by comparison, not derivation
- 3D engines that require uniform volumetric logic across scales

Here, irrational constants like π become computational artifacts, prone to approximation errors and incompatible with visual interpretation. GRM resolves this mismatch by offering a fixed-ratio alternative that aligns with how digital systems think and represent.

3.3 Comparative example: Circle area

Consider the area of a circle of radius $r = \frac{1}{2}$:

- **Classical formula**:

$$A = \pi r^2 = \pi \cdot \frac{1}{4} \approx 0.7854$$

- **GRM perspective**:

$$A = 0.7854\, SAU\ (based\ on\ s = 1)$$

Both yield the same numerical value. But GRM makes the container explicit and the comparison scalable. It reveals that a circle occupies a fixed ratio (78.54%) of the square in which it is inscribed, independent of radius or unit.

This shift supports geometry that is visual, relative, and self-normalizing.

3.4 Summary: When to use GRM

GRM is not intended to replace symbolic reasoning, it is designed for contexts where visual proportion, system consistency, and dimensional scalability are more useful than internal precision.

Table 3.4 *– Classical Geometry vs GRM*

Classical Geometry	Geometric Ratio Model
Symbolic, formula-based	Relational, ratio-based
Depends on irrational constants (e.g. π)	Uses fixed decimal ratios (e.g. 0.7854)
Radius or height driven	Container-relative
Excellent for derivation	Excellent for detection, scaling, classification
Challenging in digital systems	Naturally suited for digital logic

In short: classical geometry tells you *how* a shape is derived.
GRM tells you *how it fits, relates, and scales*, across dimensions, systems, and applications.

4. Applications of the GRM Model

The Geometric Ratio Model (GRM) is designed for practical use. By anchoring geometry in visible, scalable ratios, rather than abstract constants or internal parameters, it offers a new way to measure, detect, classify, and compare shapes across a wide variety of domains.

Rather than replacing traditional mathematics, GRM serves as an operational bridge between geometric theory and real-world systems.

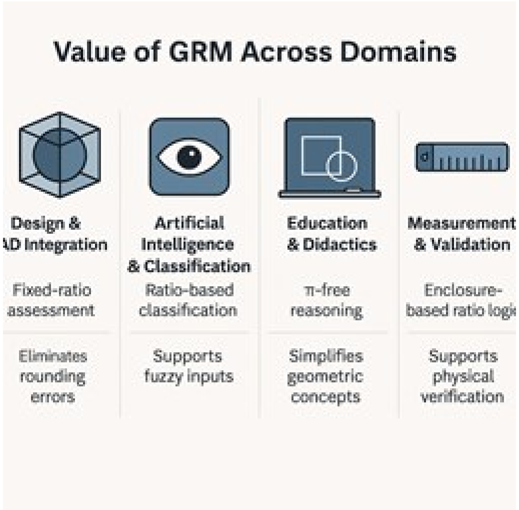

Figure 5.1 *– Applications of the GRM model*

4.1 Design and CAD integration

In computer-aided design (CAD), engineering, and 3D modeling, spatial logic is often defined by bounding boxes, visual fit, and proportionality. GRM integrates seamlessly in this context:

- Shapes can be compared based on how much of a design cell they fill (e.g. 0.5236 SVU for a sphere).
- Form validation becomes faster: does this component behave like a perfect cylinder or not?
- Patterns, tolerances, and auto-fitting tools can be calibrated to fixed ratio bands.

GRM provides a dimensionally consistent logic that extends naturally across planar, volumetric, and modular design systems.

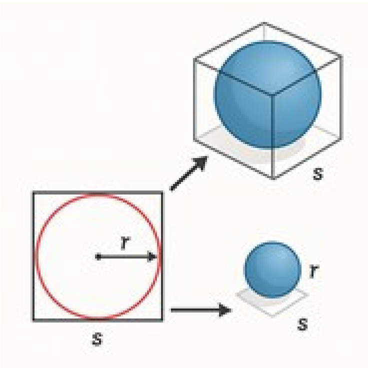

***Figure 4.2** - Design and CAD Integration*

4.2 Artificial Intelligence and classification

Shape detection in artificial intelligence, especially in image recognition and computer vision, often relies on segmentation and ratio-based logic. GRM provides a scalable, explainable metric:

- A neural network can flag a 2D shape that covers ~0.7854 of a bounding square as "likely circular".
- In 3D, voxel coverage within a cube can be compared to 0.5236 SVU to test for spherical symmetry.
- Tolerance bands allow for fuzzy classification (e.g. "circlish" or "close to sphere").

GRM enables symbol-free, structure-based classification, a powerful advantage in explainable AI (XAI) and low-power embedded systems.

Figure 4.3 – Metaverse scene.

4.3 Education and didactics

GRM supports a visual and relational understanding of geometry that is intuitive for students, especially those in technical, vocational, or early education tracks:

- No radius, π, or symbolic formulas required, just concrete shapes in fixed containers.
- Proportional reasoning becomes visible and testable using cut-outs, grids, or water.
- Students grasp how shapes relate before learning how they are derived.

This makes GRM a strong candidate for early geometry curricula, didactic tools, and comparative reasoning frameworks.

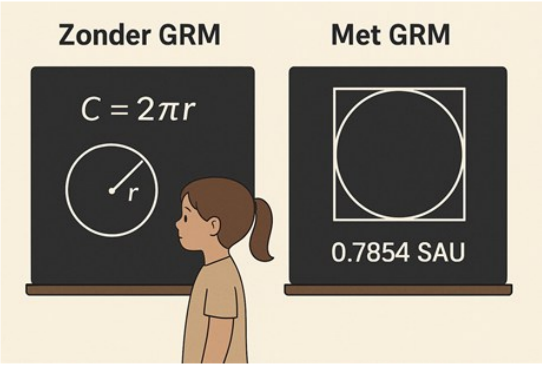

Figure 4.4 - *Geometry Education: Without vs. With GRM*

4.4 Measurement and validation

In physical environments, GRM supports non-symbolic, observable validation of geometric identity:

- A circle's "circularity" can be confirmed by its area coverage within a square.
- Physical objects can be submerged, scanned, or projected into standard frames and assessed by ratio.
- Measurement becomes about fit and proportion, not symbolic calculation.

This approach holds promise for applications in metrology, robotics, quality control, and prototyping—especially when calibration must be visual, fast, and device-agnostic.

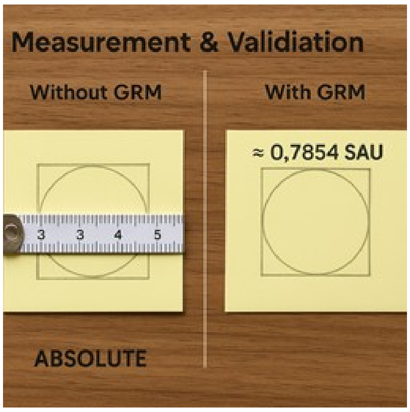

Figure 4.4 - *Measurement becomes about fit and proportion*

4.5 Summary

The GRM model excels where geometry needs to be:

- **interpreted**, not just derived;
- **observed**, not just calculated;
- **scaled**, not just symbolically defined.

It offers a single proportional language for use across design, education, artificial intelligence, digital modeling, and physical measurement.

Table 4.5 - *Overview of GRM model applications by domain*

Application Domain	Measured Value (SPU/SAU/SVU) vs. Classical Approach	Main Applications	Key Features / Noteworthy Aspects
Design & CAD Integration	GRM uses fixed ratios instead of floating-point geometries or symbolic equations	Form validation, bounding-box logic, tolerance classification	Eliminates rounding errors, improves layout validation, enables shape consistency across designs
Artificial Intelligence & Classification	GRM interprets shapes based on occupancy ratios, not parametric descriptions	Shape detection, fuzzy classification, deviation scoring	Canonical ratios provide explainable AI logic; supports imperfect or noisy inputs
Education & Didactics	Replaces π and irrational values with fixed, visual ratios within reference frames	Teaching proportions, visual logic, simplification of geometric reasoning	Enables concrete understanding from primary to technical education; π-free entry into geometry
Measurement & Validation	Uses enclosure-based ratio logic instead of abstract formulaic derivation	Physical validation, real-world ratio estimation, visual prototyping	Directly measurable using tape, tiles, or volume; supports prototyping and physical verification

5. Limitations and scope

No model explains everything. The strength of the Geometric Ratio Model lies precisely in its focus: it is not intended to replace classical geometry, but to complement it in contexts where visual structure, proportion, and digital logic are paramount. Recognizing both its power and its boundaries is essential to understanding what GRM is, and what it is not.

5.1 Where GRM excels

GRM reaches its full potential when geometry is not merely calculated, but seen. In environments where a shape’s relationship to its containing structure is more relevant than its internal measurements, GRM provides immediate clarity. A circle that fills 78.54% of a square communicates more than just its area; it reveals its identity.

This relational thinking is especially useful in systems where geometry must be interpreted or classified, such as digital modeling, AI, or education. Rather than relying on radius or angle, the GRM expresses each shape by how much of its container it occupies. This makes the model highly effective in settings where symbolic abstraction becomes a limitation, and visual measurability becomes an asset.

Whether in the classroom, in CAD, or in AI detection systems, the GRM offers a stable, scalable, and human-readable geometry that performs best when precision is matched by proportional fit.

5.2 Where GRM does not apply

Yet GRM is not a universal tool. There are domains where symbolic reasoning, continuous derivation, or irrational constants remain essential. For example, the model is not suited to integral calculus, the analysis of conic sections, or algebraic proofs involving transcendental numbers. Where geometry must be expressed through parametric forms, infinite curves, or proofs based on irrational relationships, classical mathematics remains unmatched.

GRM also does not attempt to describe free-form, chaotic, or topologically complex structures that defy enclosure within simple containers. Its logic is bounded by containment, and that is by design. The model is not meant for every shape, but for the many shapes that can be usefully understood in terms of what they occupy.

5.3 Perfect fit and practical imperfection

The logic of GRM assumes that shapes are perfectly inscribed, meaning the enclosed form touches the container at its defining boundaries. In reality, physical objects and digital shapes rarely conform with absolute perfection. Whether due to pixelation, noise, irregularity, or human error, forms often deviate slightly from the ideal.

Rather than undermining the model, these deviations open a path for further development. In GRM, difference is not failure, it is signal. A shape that almost fills 0.7854 of its container may be "circular enough" to be classified as such. This tolerance-based reasoning forms the basis of GRM's extension into classification, fuzzy logic, and AI. It accepts that identity can exist within margins, and embraces imperfection as a source of insight.

5.4 Dimensional boundaries

The GRM is currently defined within the familiar spatial dimensions: 1D, 2D, and 3D, with early explorations into 4D logic (e.g., time-aware or nested ratios). The model assumes Euclidean containment and regularity: a square contains a circle, a cube contains a sphere.

This limitation does not reduce its utility, but it sets its scope. GRM is not (yet) designed to operate in non-Euclidean geometries, curved space, or dynamic topologies. Its strength lies in fixed frames, proportional logic, and spatial consistency.

Future work may expand these boundaries, but the foundation remains dimensionally anchored: the square and the cube are not constraints, they are canvases.

5.5 Clarity within limits

Every model succeeds by choosing what to leave out. GRM succeeds by staying close to what is visible, measurable, and meaningful in real-world and digital systems. Its value is not in replacing classical formulas, but in giving designers, educators, and engineers a new geometric language, one based on relative identity, not abstract derivation.

It is not a geometry of everything. But it may well be the geometry of how we see, scale, and structure the world.

6. Future work and extensions

The Geometric Ratio Model is both complete in its core logic and open in its potential. While its foundational structure (based on fixed ratios within dimensional containers) is stable and operational, several promising directions remain to be explored. These extensions aim not to complicate the model, but to expand its utility in systems that demand flexibility, tolerance, and deeper abstraction.

The work ahead is not about redefining the model, but about unfolding its possibilities.

6.1 Toleranced classification and deviation logic

Real-world and digital shapes are rarely perfect. Whether due to imperfect rendering, noise, hand-drawn deviation, or manufacturing tolerance, most shapes deviate (slightly or significantly) from ideal forms.

To accommodate this, the GRM will be extended with tolerance bands around each canonical ratio. These bands will define not only what a circle *is*, but when a shape is *close enough* to be accepted as such. This opens the door to:

- confidence-based shape classification,
- fuzzy geometric recognition in AI,
- deviation detection in quality control,
- and probabilistic modeling in uncertain environments.

Such extensions make the GRM more resilient to imperfection, without losing its rational core.

6.2 Compound and multi-shape systems

Many practical objects do not consist of single geometric forms, but of compound assemblies: a wheel with a hub, a house with a roof, a robot arm composed of cylinders and joints. The next step in GRM development is the introduction of modular ratio logic:

- How does a shape composed of a hexagon + circle behave within a square?
- Can multiple shapes be described as a total ratio of a container?
- What is the signature of a known object (e.g., screw, chair, vertebra) in GRM terms?

This opens the model to semantic geometry: identity derived from the combination of structural proportions.

6.3 GRM for Artificial Intelligence

The combination of fixed ratios, tolerance bands, and scalable logic makes GRM a natural fit for AI-driven shape detection and classification. Future work will focus on:

- JSON schema development for ratio-based geometry,
- plugin integration with CAD, AI, and design tools,
- creation of a ratio-first dataset for supervised learning,
- and explainable AI metrics based on geometric identity.

These steps will bring GRM from theoretical model to functional toolset, ready to be implemented in digital vision pipelines.

6.4 Dimensional expansion and quantum reasoning

Although the current model focuses on 1D–3D logic, preliminary work explores GRM's potential in higher-dimensional abstraction. In quantum geometry, for example, where certainty dissolves and observation defines the system, GRM may offer a relational and observer-centric framework.

Similarly, in digital twin systems and simulation engines, time and ratio may converge into 4D proportion systems, where stability is derived not from position, but from containment and continuity.

This frontier remains speculative, but promising.

6.5 Towards a proportional geometry paradigm

What begins as a new model of measurement may evolve into a new way of thinking about geometry itself. In this vision, the square and the cube are not just practical frames, but reference bodies for relational structure. Measurement becomes not about exactness, but about how one thing fits within another, across dimensions.

As GRM continues to grow, it invites researchers, educators, developers, and designers to ask not only what something is, but how much of its space it truly occupies.

Appendix A – GRM figures and visual standards

This appendix provides key visual references that support the geometric definitions and ratio logic described in the Geometric Ratio Model (GRM). Each diagram illustrates a standardized GRM configuration based on perfectly inscribed shapes within bounding squares or cubes. The figures are intended to support practical application, validation, and didactic clarity.

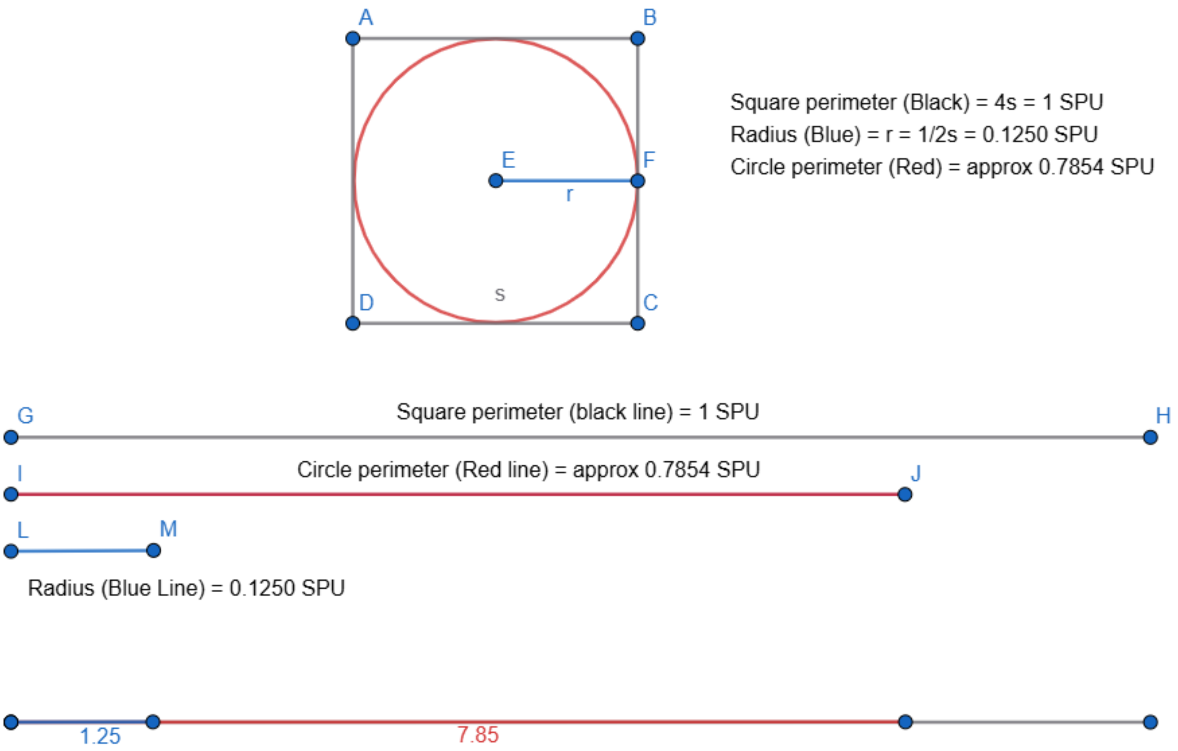

Figure A.1 — SPU Ratio and Radius Visualization for a Circle

This figure shows a square (ABCD) with an inscribed circle that touches all four sides. This configuration is the foundation for the Square Perimeter Unit (SPU) in the GRM model.

The black square has a perimeter defined as 1 SPU (4s).

The red circle has a perimeter of ≈ 0.7854 SPU (π/4 of the square).

The blue radius is exactly 0.1250 SPU, derived as r = s / 2 when the square's perimeter is 1 SPU (implying s = 0.25).

Interpretation:
These ratios are only valid when the circle satisfies the original structural definition of a GRM-conforming shape: it must be perfectly inscribed, centered, and touching all sides of the square. If these conditions are not met, the values lose geometric validity.

This visual standard reinforces the GRM's key principle: ratios represent not just numeric values, but precise shape-to-container relationships.

Appendix A.1 – Shape extensions: triangle and hexagon

The following figures illustrate two additional GRM-compatible shapes, an isosceles triangle and a regular hexagon, perfectly inscribed within a square. Each configuration adheres to the GRM principle of geometric containment and yields a fixed area ratio relative to the enclosing square. These standardized forms support proportional classification and are suitable for shape-based analysis, construction, and AI detection tasks.

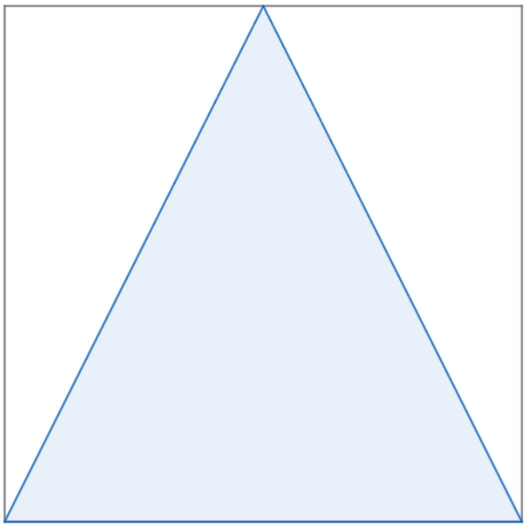

Figure A.1.1 *– isosceles Triangle Inscribed in a Square*
A perfectly centered isosceles triangle is inscribed within a square. Its base spans the full bottom edge of the square, and its apex touches the midpoint of the top edge. This configuration satisfies the GRM requirements of symmetry and structural fit, resulting in a fixed area ratio of ≈ 0.4330 SAU.

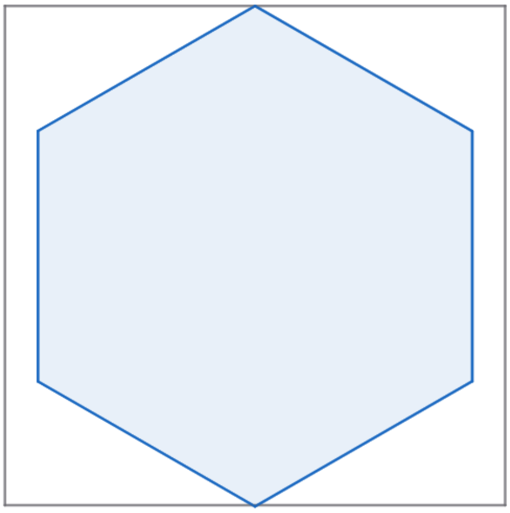

Figure A.1.2 *– Regular Hexagon Inscribed in a Square*
The regular hexagon is symmetrically placed within the square such that its top and bottom edges align horizontally, and all vertices remain within the container. This configuration preserves full symmetry and reproducibility, yielding a GRM area ratio of ≈ 0.8660 SAU.

Appendix B – The regular hexagon as an extension within the Geometric Ratio Model

B.1 Introduction

The Geometric Ratio Model (GRM) builds on standard shapes such as the square, circle, and sphere to express ratios across various dimensions. This appendix expands the model by incorporating a highly symmetrical and widely used geometric shape: the regular hexagon.

This addition does not alter the model's core structure, but rather demonstrates its extensibility to other shapes that can be inscribed within the standard framework. In the future, other polygonal or polyhedral forms may be explored similarly.

B.2 Ratios of perimeter and area

Perimeter

The perimeter of a regular hexagon is given by:

$$P_{hex} = 6a$$

where a is the side length of the hexagon. When the hexagon is inscribed within a square of side s, its optimal side length is $a = \frac{s}{\sqrt{3}}$, leading to:

$$P_{hex} = \frac{6s}{\sqrt{3}} \approx 3.464s$$

The perimeter of the square is:

$$P_{square} = 4s$$

The ratio of the hexagon's perimeter to that of the square—referred to as Hexagon-SPU—is:

$$Hexagon - SPU = \frac{P_{hex}}{P_{square}} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} \approx 0.8660$$

Area

The area of a regular hexagon is given by:

$$A_{hex} = \frac{3\sqrt{3}}{2}a^2$$

Substituting $a = \frac{s}{\sqrt{3}}$

$$A_{hex} = \frac{3\sqrt{3}}{2} = \left(\frac{s^2}{3}\right) = \frac{\sqrt{3}}{2}s^2 \approx 0.8660s^2$$

The area of the square is s^2, thus the Hexagon-SAU becomes:

$$Hexagon - SAU = \frac{A_{hex}}{s^2} = \frac{\sqrt{3}}{2} \approx 0.8660$$

B.3 Summary of ratios

The regular hexagon exhibits a unique symmetry within the GRM model: both its perimeter and area amount to approximately 86.60% of the square in which it is inscribed.

***Table B.1** – GRM Hexagon*

Quantity	Ratio Relative to the Square	Approximate Value
Perimeter	$\frac{3}{2\sqrt{3}}$	≈ 0.8660
Area	$\frac{\sqrt{3}}{2}$	≈ 0.8660

This symmetrical correspondence is noteworthy. Unlike the circle (where perimeter and area ratios differ), the hexagon maintains a consistent ratio across both dimensions. This makes it particularly useful in applications where both space coverage and edge length matter, such as in honeycombs, crystalline structures, and digital tessellations.

B.4 Future extension: The hexagonal prism

The next logical extension is to explore the hexagonal prism, a three-dimensional solid with a hexagonal base and vertical height. This form is prevalent in engineering and structural contexts. Within the GRM model, it could be compared to a cube that fully encloses it, similar to how a sphere is analyzed in relation to the cube. The volume ratio of such a prism would potentially lead to a new SVU variant, one suited to hexagonal geometry.

B.5 Conclusion

Adding the regular hexagon to the GRM model demonstrates the model's flexibility beyond circular forms. The consistent ratio of approximately 0.8660 for both perimeter and area highlights the hexagon's geometric elegance and its potential for broader application.

This appendix provides a foundation for further exploration of additional shapes and dimensions within the SPU framework. The inclusion of polygonal and polyhedral forms reinforces the model's value as a scalable and intuitive metric system.

Appendix C – Shape classification in GRM context

One of the most promising extensions of the Geometric Ratio Model is its capacity to support toleranced classification. Rather than defining shapes solely by their formulas or labels, GRM introduces a framework where shapes are understood by how much of a container they occupy, enabling graded identification even in noisy or imperfect conditions.

This approach is particularly useful in digital vision systems, AI classification, and low-resolution environments, where precise symbolic parameters (like radius or side length) may not be reliably extracted. Instead, GRM enables shape classification through ratio proximity and confidence zones.

Canonical vs. approximate identity

In the table below, we present an example **of ratio-based shape classification,** using surface ratios (SAU) for 2D shapes inscribed in a unit square.

Each row represents a different identity zone:

- ***Canonical***: the shape fits the square perfectly according to its geometric definition.
- ***Near-canonical ("-ish")***: the shape approximates the canonical value closely, often within a tolerance band (e.g., ±0.03).
- ***Other***: the ratio deviates significantly and suggests either noise, compound forms, or indeterminate identity.

***Table C.*1** - *Canonical vs. Approximate Identity*

Ratio Range (SAU)	Interpreted Shape	Classification	Notes
0.7854	Circle	Canonical	Exact fit; π/4 of square area
~0.75–0.78	Circular-ish	Approximate	Slight deformation or resolution loss
~0.72–0.74	Rounded triangle	Transitional	May suggest curved triangle or ellipse
~0.64–0.66	Hexagon	Canonical	Regular hexagon inscribed in square
~0.60–0.63	Hexagon-ish	Approximate	Asymmetric or slightly rotated hexagon
~0.42–0.44	Equilateral triangle	Canonical	60° triangle with full contact
< 0.40	Triangle-ish or partial	Degraded/Unknown	Possibly non-inscribed, incomplete or compound

Classification as confidence gradient

This logic allows for confidence scoring: a shape with a measured SAU of 0.783 is highly likely to be a circle; one with 0.751 may still be circular enough for classification, depending on context. By defining tolerance bands around canonical ratios, GRM supports flexible identity labeling, ideal for fuzzy systems, computer vision, or didactic approximation.

The model does not enforce strict binary logic but instead encourages graded interpretation. This makes it applicable in both deterministic (e.g., design validation) and probabilistic (e.g., AI detection) contexts.

Future extensions

This appendix represents an early example of GRM-based classification logic. In whitepaper E-1, we explore how these tolerances can be formalized, embedded in systems, and connected to use cases in manufacturing, pattern recognition, and shape-based retrieval.

GRM thus offers not only a language for geometric identity, but also a logic for structural resemblance and deviation.

Appendix D – Related models and inspiration

While the Square Perimeter Unit (SPU) model offers a unique and independently developed framework, it is part of a broader movement in geometry that seeks alternative approaches to describing shapes, proportions, and circular structures without relying on the traditional use of π. The following sources illustrate similar ideas or complementary lines of thought:

Hartl, M. (2010). *The Tau Manifesto*

A manifesto advocating for the use of the constant τ (tau = 2π) as a more intuitive alternative to π. Hartl argues that many mathematical formulas become simpler and more elegant when τ is used, particularly in circular geometry.

Website: www.tauday.com

OSF Preprints (2023). *A Circle Without Pi*

This preprint introduces an alternative framework for understanding the circle, emphasizing ratio-based relationships without the use of π. The focus is on visual and relational properties of circles within fixed geometric containers.

Link: https://osf.io/preprints/osf/stwxf

Monte Carlo Approaches to Estimating Circle Area

In probabilistic mathematics, it is possible to estimate the area of a circle without directly using π. Monte Carlo simulations, where random points are placed inside a square, can statistically approximate π as a ratio of filled space.

See, for example, related discussions on Reddit Math that explore various methods.

Reflection

These sources demonstrate that the idea of replacing or reformulating π is not isolated, but part of a broader exploration of simplicity, intuition, and measurability in geometry. The GRM model aligns with this movement by offering a systematic, visual, and scale-free approach centered on fixed ratios within standard shapes such as the square and the cube.

The model distinguishes itself by emphasizing didactic clarity, physical reproducibility, and dimensional consistency, making it a bridge between classical mathematics and modern practical applications.

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