WHITE PAPER

The Geometric Ratio Model (GRM)

A New Metric for Proportion, Shape, and Dimension

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Summary

This document introduces the GRM model as a scale-free, visually robust, and dimensionally consistent approach to geometry.

The model replaces abstract irrational constants, such as π, with fixed ratios within standard shapes (square and cube) and offers applications in education, engineering, visualization, design, and systems thinking.

It is both didactically accessible and theoretically scalable to higher dimensions.

Inhoud

1. Introduction and Core Vision

1.1 Background

The mathematical constant π (pi) has served as the cornerstone for calculating the circumference and area of circles for centuries. While its definition as the ratio of a circle's circumference to its diameter is mathematically indisputable, its application in education, geometry, and digital contexts presents inherent limitations: it is an irrational number that cannot be precisely noted in decimal form and is only indirectly measurable. For many, π remains an abstract concept, disconnected from tangible reality.

1.2 Purpose and Motivation

The purpose of this GRM model is to propose an alternative to the use of π as the foundational constant in geometry by describing shapes in relation to a concrete, universally measurable standard: the line in a square (for 1D), the plane of the square (for 2D), and the volume of the cube (for 3D). The proposed approach uses these standards as reference units for circumference, area, and volume.

The core concept is both simple and powerful: When a circle is precisely inscribed within a square, the ratio of the circle's circumference to that of the square is exactly π/4 ≈ 0.7854. This ratio is no longer regarded as a coincidental consequence of π but as a definable unit: the Square Perimeter Unit (SPU). Similarly, the circle’s area and the sphere’s volume can be expressed as fixed ratios relative to the square’s area (Square Area Unit, SAU) and the cube’s volume (Square Volume Unit, SVU).

1.3 Core Vision

The GRM model proposes that geometric shapes no longer be described exclusively in absolute units or via irrational constants, but through relative ratios within a fixed metric structure. This results in a scale-free, consistent, and dimensionally extendable system where:

- The circumference of a circle is expressed as π/4 ≈ 0.7854 SPU
- The area of the same circle is expressed as π/4 ≈ 0.7854 SAU
- The volume of an inscribed sphere is expressed as π/6 ≈ 0.5236 SVU

This model is not intended as a replacement for π in classical mathematics but as a practical approach within educational, digital, and geometric contexts where practical proportions and measurability are paramount. By reformulating π as a derived ratio within a measurable standard, this model allows for a new perspective on shape, volume, and proportion.

Positioning within the geometric tradition
The Square Perimeter Unit model is rooted in classical geometric principles but reformulates them in a way that meets modern demands for simplicity, scalability, and measurability. By moving away from abstract constants like π and instead using fixed ratio units within standard shapes, it offers a new perspective on geometry that is both visually intuitive and practically applicable.

1.4 Outlook on Higher Dimensions

Although the GRM model primarily focuses on the first three spatial dimensions in this document, there is a clear opportunity for extension into higher dimensions. The ratios between inscribed hyperspheres and hypercubes are similarly definable, measurable (via analogies), and consistent within the VE-n framework. Thus, the model has the potential to evolve into a universal, dimensionally scalable measurement system that transcends classical space.

The following chapters will elaborate on this model step by step, tested through practical applications, geometric considerations, and theoretical reflections.

2. Foundations of the GRM Model

2.1 The Standard Form: The Square and the Cube

*In this chapter, the notation s is used to denote the length of one side of a square or cube. This convention is common in geometry and serves as the basis for the standard formulas within the GRM model.

The cornerstone of the GRM model is the selection of the square (in 2D) and the cube (in 3D) as universal measurement units. These shapes are not only simple to construct and measure, but also function as natural frameworks within which other geometric forms can be enclosed. The square structure enables exact and consistent proportions to be defined for measurements of length (1D), area (2D), and volume (3D).

In the GRM model:

- The perimeter of the square is defined as 1 SPU
- The area of the square is defined as 1 SAU
- The volume of the cube is defined as 1 SVU

These units serve as reference points for all relative measurements within this model.

Overview Table: Standard Ratios in the GRM Model

Dimension	**Measurement Aspect**	**Standard Shape**	**Formula** (s=1)	**Ratio**	**Unit**
1D	Length (perimeter)	Square	$4s$	1	1 SPU
2D	Area	Square	s^2	1	1 SAU
3D	Volume	Cube	s^3	1	1 SVU

Supplementary Overview: Ratios in Standard and Inscribed Shapes

The table below provides fixed ratios between standard shapes (square, cube) and their inscribed forms (such as circle and sphere), with a side length of s = 1. The values are normalized relative to the square or cube enclosing the form. This overview enhances understanding of how SPU, SUA, and SVU function as universal measurement ratios within the model.

Dimension	**Measurement Aspect**	**Standard Shape**	**Formula (where s = 1)**	**Ratio**	**Unit**
1D	Perimeter (Length)	Square	$4 . s = 4$	1	1 SPU
2D	Area	Square	$s^2 = 1$	1	1 SAU
3D	Volume	Cube	$s^3 = 1$	1	1 SVU
2D	Circle (Inscribed)	Circle	$\pi . d = \pi \ (d = s = 1)$	$\pi/4 \approx$ 0.7854	SPU/SAU
3D	Sphere (Inscribed)	Sphere	$\frac{4}{3}\pi r^3 \ (r = 0{,}5)$	$\pi/6 \approx$ 0.5236	SVU

2.2 Enclosed Shapes as Fixed Ratio Carriers

When a geometric shape is perfectly inscribed within the standard form (square or cube), it generates fixed proportions independent of absolute dimensions. For a circle with a diameter equal to the side length of the square:

- Circle perimeter: $C = \pi d = \pi s$
- Square perimeter: $C = 4s$
- Ratio: $\pi/4 \approx 0.7854$ SPU

The same logic applies to area and volume:

- Circle area relative to square: $\pi/4 \approx 0.7854$ SAU
- Sphere volume relative to cube: $\pi/6 \approx 0.5236$ SVU

This enables circles, spheres, and other shapes to be compared geometrically and metrically without relying on irrational numbers.

Note: In this context, s equals the diameter of the circle, as the circle is perfectly inscribed within the square. The formula $C = \pi s$ is correct as long as s is interpreted as both the side length of the square and the diameter of the circle. In situations where s traditionally represents the radius, the formula would naturally become $C = 2\pi s$.

2.3 Dimensional Consistency: 1D–2D–3D

The strength of the GRM model lies in its internal consistency across dimensions:

- In 1D, the perimeter of the square forms the baseline: any linear structure can be related to the square perimeter.
- In 2D, the area of the square serves as the comparative plane.
- In 3D, the volume of the cube becomes the benchmark for volumetric ratios.

Since circles, spheres, and other shapes are consistently placed within the same standard framework, a coherent system of ratios emerges that remains consistent across all dimensions.

2.4 Measurability and Visualizability

Unlike π, which is an abstract and immeasurable constant, the ratios within the GRM model can be visually and physically approximated. They can be experimentally determined using measuring tapes, surface tiles, or water displacement (for volume). This makes the model not only suitable for theoretical applications but also for educational purposes, didactics, and digital shape analysis.

2.5 Precision and Error Margins

Practical Accuracy within the GRM Model

One of the core advantages of the Geometric Ratio model (GRM) is its use of fixed ratio units—such as SPU, SAU, and SVU—instead of irrational constants like π. While π is represented in classical geometry as an infinite, non-repeating decimal, the GRM model uses rounded values that are more than sufficiently accurate for applications in education, design, visualization, and engineering.

The key ratios used in the GRM model are:

- $\frac{\pi}{4} \approx 0.7854$ → **SPU / SAU** (for the circumference and area of an inscribed circle)
- $\frac{\pi}{6} \approx 0{,}5236$ → **SVU** (for the volume of an inscribed sphere)

Though these values are approximations, the actual absolute error in practical use is negligibly small.

Comparison: Classical π vs. SPU Ratios

Example 1: Circumference of a circle inscribed in a square (s = 1)

Method	Formula	Value	Absolute Error
Classical	$C = \pi s$	3.1416	—
GRM model	$C = 4 \cdot 0.7854$	3.1416	≈ 0.0000

Identical to four decimal places. The error appears only after the fifth decimal.

Example 2: Area of an inscribed circle

Method	Formula	Value	Absolute Error
Classical	$A = \pi s^2/4$	0.7854	—
GRM model	$A = 0.7854 \cdot s^2$	0.7854	0

Numerically identical in practical terms.

Example 3: Volume of a sphere inscribed in a cube (s = 1)

Method	Formula	Value	Absolute Error
Classical	$V = \pi s^3/6$	0.5236	—
GRM model	$V = 0{,}5236 \cdot s^2$	0.5236	0

What Does This Offer?

1. **Faster Calculations**

   By using fixed ratios, there's no need for π-based calculations or approximations. A single multiplication factor suffices—saving time and minimizing human or computational error.

2. **Simplification in Education and Training**

   Students and learners can work with concrete values instead of abstract constants. This makes geometry more approachable, especially in primary education or technical training contexts.

3. Consistency in Digital Systems

Fixed ratio values like 0.7854 (SPU/SAU) and 0.5236 (SVU) can be directly implemented in programming, CAD systems, or AI models—without floating-point errors from irrational constants.

4. Real-World Measurability

The ratios are physically verifiable using measuring tape, surface tiles, or water displacement. This bridges the gap between mathematical models and the tangible world—ideal for experimentation, physical prototyping, or didactic demonstrations.

Conclusion

Although the GRM model uses rounded ratio units rather than formal mathematical constants, the resulting deviation in practical applications is negligibly small. The error margins lie well within tolerances used in education, design, and engineering. This makes the model a powerful alternative for contexts where clarity, reproducibility, and consistency matter more than mathematical perfection to the nth decimal. By combining simplicity with practical precision, the GRM model provides a robust and efficient metric for geometric reasoning.

3. Comparison with the Classical π Approach

3.1 The Role of π in Classical Geometry

The mathematical constant π is defined as the ratio between the circumference of a circle and its diameter. It is an irrational number, with a non-terminating and non-repeating decimal representation, approximately equal to 3.14159. In classical geometry, π is indispensable for calculating the circumference and area of circles and plays a central role in the analysis of spheres, cylinders, cones, and other rounded forms.

While mathematically exact, π poses certain limitations in practical applications:

- It cannot be expressed exactly in decimal notation.
- Its use necessitates rounded approximations for numerical computations.
- It is abstract and not directly measurable in physical reality.

These characteristics render π less accessible in didactic environments and in contexts requiring simplicity and measurability, such as digital frameworks.

3.2 The SPU Perspective: Ratio Instead of a Constant

The GRM model reframes the problem by replacing the abstract constant with a concrete ratio within a standardized form. For a circle inscribed within a square, the following applies:

SPU = circumference of circle / circumference of square = πs / 4s = π / 4 ≈ 0.7854

$$SPU = \frac{circumfence\ of\ cricle}{circumfence\ of\ square} = \frac{\pi s}{4s} = \frac{\pi}{4} \approx 0.7854$$

The traditional value of π is thus reduced to a ratio within a measurable structure. This substitution transforms π from a constant into a direct relationship between an enclosed shape and its surrounding framework. This shift provides the following advantages:

- It is measurable (using tape, software, simulation, or physical models).
- It is comparable (across different shapes within a single standard).
- It is dimensionally consistent (extendable to area and volume).
- It is didactically transparent (students observe the ratio rather than an abstract symbol).

3.3 Considerations and Limitations

While the GRM model offers significant benefits, it is essential to recognize that it does not replace π as a universal mathematical constant. Classical mathematics and analytical geometry remain fundamentally based on π.

The GRM model should be understood as:

- A practical reformulation for specific contexts.
- An alternative representation of ratios rather than a substitute for π.
- Most effective in scenarios requiring visual, physical, or relational interpretations of geometry.

3.4 Conclusion

The comparison with the classical π approach demonstrates that the GRM model is a valuable addition to the mathematical toolkit. By emphasizing ratios within measurable frameworks, the model proposes a novel approach to geometry—one that prioritizes relational measurability over abstract constants. This makes it particularly suitable for educational purposes, digital applications, and multidimensional analyses.

4. Applications of the GRM Model

4.1 Education and Didactics

The GRM model provides an accessible and visually compelling way to concretize abstract concepts such as circumference, area, and volume. By consistently relating shapes to a square or cube with known dimensions, it creates a natural context for understanding and comparison. Rather than familiarizing students with an irrational constant like π, the model allows them to observe, measure, and comprehend ratios within a tangible framework.

This approach aligns with educational principles such as experiential learning and visual reasoning. For example, a lesson could be designed where students cut out circles, fit them into squares, and then measure the ratio of the circumference to the square using a string. For understanding volume, a sphere can be submerged in a measuring cylinder or water tank to experimentally determine its ratio to the cube's volume. In this way, abstract geometry is transformed into experiential understanding.

Added Value Compared to the Classical Approach:

While classical geometry is rooted in abstract constants like π, the GRM model offers an alternative grounded in relative, directly measurable proportions within a fixed framework. This not only allows for a mathematical understanding but also enables a visible and tangible structure. By using standard shapes such as the square and the cube as references, the method becomes more intuitive, visually compelling, and better suited to practical contexts such as education, simulation, design, and data analysis.

Didactic Anchoring in Educational Principles:

The GRM model integrates well into educational frameworks emphasizing experiential and relational learning. It aligns with holistic education, where knowledge is constructed through sensory and meaningful interaction with the world, and with constructivist approaches, where students build knowledge by exploring and comparing. The model also complements modern STEM/STEAM approaches, which connect mathematics to technology, art, and design. This makes it a robust foundation for integrated learning, engaging spatial reasoning, logic, and creativity.

Educational Applications:

- Circle comprehension without π: students measure the circumference of a circle and the square in which it fits.
- Visualizing area and volume as fixed ratios within a standard shape.
- Developing didactic materials based on SPU/SAU/SVU to enhance conceptual understanding.

4.2 Digital and Geometric Applications

In digital environments, CAD software, and graphic simulations, systems often operate on raster, grid, or pixel-based frameworks. The GRM model seamlessly integrates into these systems as it employs fixed, measurable units and relative proportions.

In digital applications involving shape recognition, simulation, or CAD design, the GRM model allows geometry to be addressed not in absolute pixels or millimeters but relative to a standard shape. For instance, an AI algorithm may classify objects based on their relative 'filling' of a square (SPU) or a cube (SPU), regardless of their absolute dimensions. Graphic engines can also use SPU to scale or anchor shapes within grid structures automatically, enhancing accuracy and consistency in user interfaces or game design.

Added Value Compared to Current Methodology:

Most digital systems approach shapes through absolute coordinates or pixels, without explicitly considering their relative geometric structure. This limits the scale-transferability of shape recognition across contexts. The GRM model introduces normalization, where proportions within a fixed standard (square or cube) take precedence, making shapes scale-independent, system-neutral, and dimensionally comparable. This improves the robustness, generalizability, and intuitiveness of algorithms, especially in AI applications such as object recognition or pattern analysis.

Potential Applications in Digital Contexts:

- Shape recognition through fixed proportions (e.g., AI-based geometric classification).
- Scale-free comparisons between objects in 2D or 3D renders.
- Interactive measurement modules where objects are always placed within a square or cube.

4.3 Design, Engineering, and Visualization

In technical domains such as product design, engineering, and construction, the ability to relate shapes to standard dimensions is crucial. The GRM model provides a consistent framework for this purpose.

In design and engineering, speed and precision are essential. The GRM model enables designers to quickly compare complex objects to a standard shape. For example, an industrial designer might express the ratio between a spherical reservoir and the cube in which it fits in SPU to gain immediate insight into residual volume or utilization rate. In visualization processes, the difference between a square and an inscribed circle (residual shape) can be used to optimize material efficiency or generate artistic forms with consistent proportional structures.

Added Value Compared to Current Methodology:

In many design and engineering environments, shapes are evaluated based on absolute dimensions, areas, or volumes, often using traditional units like millimeters or cubic centimeters. This approach requires context-specific interpretation and is less suitable for quick relative comparison. In contrast, the GRM model enables direct work with proportions relative to a fixed standard, allowing for faster, more visual decision-making. This lowers cognitive load and facilitates communication between designers, engineers, and clients about scale, utilization, or shape efficiency.

Applications in Design and Engineering:

- Faster ratio analyses during prototyping and scale modeling.
- Ability to relate complex shapes to easily constructed base dimensions.
- Visualization of residual shapes and content distribution based on differences with the standard.

4.4 Interdisciplinary Potential

The GRM model is not limited to mathematical or geometric applications.

Since the model is based on the ratio between a shape and its enclosing standard, it offers a universal principle applicable beyond classical geometry. In visual arts, designers can create compositions where shapes occupy an exact percentage of a square or cube, leading to harmony or tension in a piece. In physics, the model provides a framework for quantifying relative filling (e.g., in molecular structures or packing density). In philosophy or systems theory, SPU can serve as a metaphor for bounded systems where entities are analyzed based on their relationship to the whole in which they operate.

Added Value Compared to Current Methodology:

Many disciplines work with abstract models or relative approaches but lack a clear, geometrically anchored standard to frame proportions, structures, or relationships. The GRM model fills this gap by offering a consistent, visually, and computationally manageable reference framework that is both quantitatively and qualitatively useful. This enables the integration of geometric insights into systems thinking, aesthetic form analysis, or scientific interpretations of spatial relationships—without reliance on irrational constants or context-dependent measurement systems.

Because it is rooted in measurable frameworks and proportions, the GRM model is relevant for:

- Art and design (shape composition within fixed frameworks).
- Physics and chemistry (relative filling degrees).
- Philosophy and systems thinking (relational reasoning within fixed structures).

4.5 Summary

The GRM model offers a novel way of considering geometric shapes: not as isolated objects but as relationships within a standardized framework. This results in a versatile method suitable for education, simulation, design, and analysis. The exploration of applications in this chapter demonstrates that the model is not only mathematically robust but also didactically and technically valuable.

Future phases will focus on translating these insights into concrete implementations in educational modules, digital tools, and design systems. Methodological and didactic justification for this translation will also be addressed.

4.6 Table: Overview of GRM model applications by domain

Application Domain	Measured Value (SPU/SAU/SVU) vs. Classical Approach	Main Applications	Key Features/Noteworthy Aspects
Education	Fixed proportions instead of irrational numbers like π	Experiential measurement, didactic materials, visual reasoning	Aligns with holistic, constructivist, and STEAM education
Digital Systems	Normalization within a standard shape instead of absolute coordinates	Shape recognition, CAD, scale-free comparisons	Provides scale-independent classification and AI compatibility
Engineering & Design	Relationship to cube or square as a standard for content and shape	Residual volume, material efficiency, visual design	Simplifies communication and optimization during prototyping
Art & Design	Composition based on fixed proportions relative to canvas or space	Geometric balance, aesthetics, visual rhythm	Can be applied intuitively without mathematical expertise
Systems Thinking & Philosophy	Relationships within bounded systems (metaphor)	Structural models, relational reasoning	Suitable for metaphorical or model-based approaches to reality

5. Deepening, Reflection, and Future Vision

5.1 Reflection on the model and its principles

The GRM model emerged from the desire to reformulate geometric proportions in a manner that is both concretely measurable and didactically comprehensible. By moving away from abstract constants such as π and instead utilizing fixed ratios within standard shapes (square, cube), the approach anchors itself in physical reality while remaining accessible for diverse applications. Reflection on the core principles reveals that the model is consistent, scale-independent, and dimensionally extensible without losing its simplicity.

At the same time, it is important to acknowledge its limitations: the model is not intended to replace classical mathematics but to serve as an additional method of reasoning and application. The strength lies in its relational and measurable nature, rather than absolute precision or formal generalization. The value of the GRM model, therefore, resides in its applicability within contexts emphasizing insight, overview, and comparative understanding.

5.2 Research Questions and Areas of Exploration

The model raises several questions that warrant further mathematical and didactical investigation:

- How robust are the fixed ratios (e.g., $\frac{\pi}{4}$ and $\frac{\pi}{6}$)) in applications to complex shapes?
- To what extent is the model applicable within non-Euclidean or digital geometries?
- Can SPU, SAU, and SVU also serve as alternative vector representations in AI models?
- How can educational modules be designed to enable students to discover SPU ratios on their own?

These questions serve as a foundation for further theoretical and practical elaboration of the model.

5.3 Toward a Universal Metric

The GRM model offers opportunities for extension into higher dimensions (SB-n). This potential paves the way for a unified description of enclosed shapes within standard structures, independent of dimension level. By calculating the fixed ratio between hyperspheres and hypercubes, a continuum of measurement units emerges based on fixed structures rather than irrational constants.

This approach could lead to an alternative metric that is:

- Scale-independent;
- Widely applicable within digital analysis, simulation, and systems thinking;
- Understandable to both technical and non-technical users.

Note: The model can be extended to higher dimensions. For example, the ratio of an enclosed 4D sphere relative to a 4D hypercube equals $\frac{\pi^2}{32} \approx 0.3084$. This ratio is referred to as VE4 and forms the logical continuation of SPU (1D), SAU (2D), and SVU (3D).

5.4 Implementation Prospects

In future iterations, research will focus on integrating the GRM model into:

- Educational modules for primary and secondary education;
- Calculation tools and CAD systems;
- AI models centered on shape recognition;
- Visualization and communication tools for design and analysis.

These applications require interdisciplinary collaboration, spanning fields such as mathematics, engineering, educational science, design, and computer science. A phased implementation — starting with pilot projects in education and simple software tools — appears a logical first step.

5.5 Concluding Remarks

The GRM model offers an original, visually compelling, and substantively robust approach to geometry. By reformulating abstract relationships into concrete ratios within standard shapes, geometry becomes not only more comprehensible but also more applicable. Its strength lies in its simplicity, consistency across dimensions, and potential for expansion. In an era where education, technology, and digital systems increasingly demand integration and intuition, the GRM model may contribute to a new way of thinking about shape, proportion, and measurement.

Future Additions

This initial version of the white paper has intentionally limited illustrations, tables, and visualizations. In a future edition, supportive figures will be added to visually clarify the fixed ratios within the GRM model. These will include representations of the SPU, SAU, and SVU principles, as well as additional forms like the hexagon and potential 3D extensions.

Appendix A – The Regular Hexagon as an Extension Within the Geometric Ratio Model

A.1 Introduction

The Geometric Ratio Model (GRM) builds on standard shapes such as the square, circle, and sphere to express ratios across various dimensions. This appendix expands the model by incorporating a highly symmetrical and widely used geometric shape: the regular hexagon.

This addition does not alter the model's core structure, but rather demonstrates its extensibility to other shapes that can be inscribed within the standard framework. In the future, other polygonal or polyhedral forms may be explored similarly.

A.2 Ratios of Perimeter and Area

Perimeter

The perimeter of a regular hexagon is given by:

$$P_{hex} = 6a$$

where a is the side length of the hexagon. When the hexagon is inscribed within a square of side s, its optimal side length is $a = \frac{s}{\sqrt{3}}$, leading to:

$$P_{hex} = \frac{6s}{\sqrt{3}} \approx 3.464s$$

The perimeter of the square is:

$$P_{square} = 4s$$

The ratio of the hexagon's perimeter to that of the square—referred to as Hexagon-SPU—is:

$$Hexagon - SPU = \frac{P_{hex}}{P_{square}} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} \approx 0.8660$$

Area

The area of a regular hexagon is given by:

$$A_{hex} = \frac{3\sqrt{3}}{2}a^2$$

Substituting $a = \frac{s}{\sqrt{3}}$

$$A_{hex} = \frac{3\sqrt{3}}{2} = \left(\frac{s^2}{3}\right) = \frac{\sqrt{3}}{2}s^2 \approx 0.8660s^2$$

The area of the square is s^2, thus the Hexagon-SAU becomes:

$$Hexagon - SAU = \frac{A_{hex}}{s^2} = \frac{\sqrt{3}}{2} \approx 0.8660$$

A.3 Summary of Ratios

The regular hexagon exhibits a unique symmetry within the GRM model: both its perimeter and area amount to approximately 86.60% of the square in which it is inscribed.

Quantity	Ratio Relative to the Square	Approximate Value
Perimeter	$\frac{3}{2\sqrt{3}}$	≈ 0.8660
Area	$\frac{\sqrt{3}}{2}$	≈ 0.8660

This symmetrical correspondence is noteworthy. Unlike the circle (where perimeter and area ratios differ), the hexagon maintains a consistent ratio across both dimensions. This makes it particularly useful in applications where both space coverage and edge length matter—such as in honeycombs, crystalline structures, and digital tessellations.

A.4 Future Extension: The Hexagonal Prism

The next logical extension is to explore the hexagonal prism, a three-dimensional solid with a hexagonal base and vertical height. This form is prevalent in engineering and structural contexts. Within the GRM model, it could be compared to a cube that fully encloses it, similar to how a sphere is analyzed in relation to the cube. The volume ratio of such a prism would potentially lead to a new SVU variant—one suited to hexagonal geometry.

A.5 Conclusion

Adding the regular hexagon to the GRM model demonstrates the model's flexibility beyond circular forms. The consistent ratio of approximately 0.8660 for both perimeter and area highlights the hexagon's geometric elegance and its potential for broader application.

This appendix provides a foundation for further exploration of additional shapes and dimensions within the SPU framework. The inclusion of polygonal and polyhedral forms reinforces the model's value as a scalable and intuitive metric system.

Appendix B – Related Models and Inspiration

While the Square Perimeter Unit (SPU) model offers a unique and independently developed framework, it is part of a broader movement in geometry that seeks alternative approaches to describing shapes, proportions, and circular structures without relying on the traditional use of π. The following sources illustrate similar ideas or complementary lines of thought:

1. **Hartl, M. (2010).** ***The Tau Manifesto***

A manifesto advocating for the use of the constant τ (tau = 2π) as a more intuitive alternative to π. Hartl argues that many mathematical formulas become simpler and more elegant when τ is used, particularly in circular geometry.

Website: www.tauday.com

2. **OSF Preprints (2023).** ***A Circle Without Pi***

This preprint introduces an alternative framework for understanding the circle, emphasizing ratio-based relationships without the use of π. The focus is on visual and relational properties of circles within fixed geometric containers.

Link: https://osf.io/preprints/osf/stwxf

3. **Monte Carlo Approaches to Estimating Circle Area**

In probabilistic mathematics, it is possible to estimate the area of a circle without directly using π. Monte Carlo simulations—where random points are placed inside a square—can statistically approximate π as a ratio of filled space.

See, for example, related discussions on Reddit Math that explore various methods.

Reflection

These sources demonstrate that the idea of replacing or reformulating π is not isolated, but part of a broader exploration of simplicity, intuition, and measurability in geometry. The GRM model aligns with this movement by offering a systematic, visual, and scale-free approach centered on fixed ratios within standard shapes such as the square and the cube.

The model distinguishes itself by emphasizing didactic clarity, physical reproducibility, and dimensional consistency, making it a bridge between classical mathematics and modern practical applications.

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