Proportional Design with the GRM

A Construction Framework Based on Bounding Squares and Cubes

Author:
M.C.M. van Kroonenburgh, MSc
Heerlen, The Netherlands

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Abstract

This proposal introduces a proportional design framework based on the Geometric Ratio Model (GRM), where all shapes are defined as ratios within a bounding square or cube. Instead of relying on irrational constants or internal measurements, shapes are constructed and interpreted using fixed ratios such as 0.7854 (SAU for a circle) or 0.5236 (SVU for a sphere). The model enables precise, scalable, and visually grounded shape construction across disciplines—ideal for CAD, education, UI design, and algorithmic geometry. GRM transforms geometry from formula-based calculation into a universal language of proportion.

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1. Introduction

In geometry, design, and digital visualization, shapes are traditionally defined using absolute dimensions or mathematical constants—most notably π. While effective in classical contexts, such methods often complicate design workflows when scalability, cross-dimensional consistency, or digital implementation is required.

The Geometric Ratio Model (GRM) proposes an alternative: rather than defining a shape from its internal properties (like radius or side length), every form is derived from a bounding square (2D) or bounding cube (3D). Within this framework, each geometric figure is constructed or analyzed in proportion to this standard container. These proportions are then expressed using fixed, dimensionless units—SPU (Square Perimeter Unit), SAU (Square Area Unit), and SVU (Square Volume Unit).

This approach enables forward design (e.g., drawing a shape of a desired size) and reverse analysis (e.g., estimating area or volume) without relying on irrational constants. The model provides a scalable, resolution-independent system that is both visually intuitive and computationally robust.

This proposal introduces a practical design framework based on GRM logic—one that is suitable for CAD, digital modeling, educational applications, and visual simulation environments. It outlines how any geometric figure can be embedded, constructed, or measured proportionally within a bounding square or cube, unlocking a new methodology for shape definition, transformation, and interpretation.

2. Problem Statement

Traditional geometric design relies on absolute metrics—lengths, diameters, radii, and angles—often requiring irrational constants and multiple formulaic steps to define or analyze a shape. While mathematically valid, these methods introduce several practical limitations when applied to modern design, visualization, or computational systems:

- **Inflexibility in scaling**: A shape defined by radius or area cannot be easily resized or reoriented without recalculating all dependent values.
- **Dependency on π and irrational constants**: Circles, spheres, and other curved forms rely on approximated values, leading to rounding errors and reduced clarity in instructional or digital contexts.
- **Disconnect between shape and space**: Most approaches treat shapes as independent objects, rather than as proportions relative to the space they occupy.

- **Inefficiency in software and CAD systems**: Modern design tools often require complex transformations to convert between real-world dimensions and system-specific units or grids.

Moreover, when a designer wants to draw a circle with a desired area, or estimate the proportion a shape occupies, no standard proportional framework exists to support that operation without reverting to classical formulas.

This results in a gap between conceptual design and measurable, reproducible geometry. Especially in digital and educational environments, where clarity, visual correspondence, and dimensionless logic are critical, the absence of a proportional construction model hinders both usability and interoperability.

The need is clear: a universal method to define and construct any shape through ratios, within a known bounding structure, that is:

- conceptually simple,
- mathematically exact,
- and visually and digitally intuitive.

The GRM-based approach introduced in this proposal offers such a method.

2.1 Illustrative Example – Designing a Circle with a Target Area

Consider a designer working in a CAD environment who needs to draw a circle with an exact area of 12 cm^2. Using classical geometry, this requires multiple steps involving π:

- First, compute the radius:

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{12}{\pi}} \approx 1.95$$

- Then derive the diameter:

$$d = 2r \ \approx 1.95$$

- Finally, input the circle using this diameter, trusting that the software's internal approximations of π maintain sufficient precision.

While mathematically sound, this method relies on an irrational constant (π), is non-intuitive in design workflows, and does not visually relate the shape to its spatial context.

Now compare this with the GRM-based approach:

- The circle is constructed within a bounding square.
- Since a perfectly inscribed circle occupies exactly 0.7854 SAU (Square Area Units), the required square area is calculated as:

$$A_{square} = \frac{12}{0.7854} \approx 15.28\ cm^2$$

- From this, the side of the square is easily found:

$$s = \sqrt{15.28} \approx 3.91$$

- The circle is then drawn inside this square, no further conversion needed.

This approach provides:

- A fixed ratio (0.7854) in place of an irrational constant,
- A visual, intuitive reference frame for both design and communication,
- And full compatibility with pixel-based or vector-based tools, as used in digital fabrication, UI design, and educational software.

Such proportional reasoning can be applied to any shape where a known GRM ratio exists, and enables both forward (constructive) and reverse (analytical) reasoning—simplifying workflows across disciplines.

3. Innovative Concept: GRM-Based Construction Logic

At the heart of the GRM approach lies a simple yet powerful principle: every geometric shape is interpreted as a proportion of its bounding square (2D) or cube (3D). Rather than constructing a shape from its internal parameters, such as radius or edge length, the shape is *defined* by how much of the enclosing form it occupies.

This leads to a fundamental shift in design logic:

- A circle is no longer described by its radius, but by the fact that it fills exactly 0.7854 SAU of its bounding square.
- A sphere becomes an object that fills 0.5236 SVU of its bounding cube.
- A regular hexagon, similarly, consistently occupies 0.8660 SAU/SPU, offering a symmetric ratio across perimeter and area.

These fixed values—GRM ratios—act as universal constants for shape identity and proportionality.

3.1 Forward Construction (Designing from Proportion)

When a designer knows the intended size of a shape—such as a target area or volume—they can:

- Select the appropriate GRM ratio for the intended shape (e.g., 0.7854 for a circle).
- Compute the required area (or volume) of the bounding square or cube by dividing the target value by the GRM ratio.
- Derive the side length from that square or cube.
- Construct the shape *within* the bounding structure—ensuring visual harmony and mathematical accuracy.

This process eliminates irrational constants, supports exact reproducibility, and enhances compatibility with vector design tools, 3D modeling software, and even physical drafting.

3.2 Reverse Analysis (Measuring from Occupation)

The GRM logic also enables reverse interpretation:

- When a shape is present (in a drawing, image, or physical object), its proportion relative to a bounding square or cube can be measured.
- By comparing the occupied fraction to known GRM ratios (e.g., 0.5236 SVU), its identity, size, or classification can be inferred.

This technique is especially useful in:

- Digital shape recognition
- Educational analysis
- Design validation
- AI post-processing and geometric classification

It allows for dimensionless, resolution-independent measurement through relational reasoning rather than formulaic inference.

3.3 Geometric Validity Conditions

It is essential to emphasize that GRM ratios such as 0.7854 SAU (for a circle) or 0.5236 SVU (for a sphere) are only valid under one critical condition: the shape must be perfectly inscribed within its bounding square or cube.

For a circle, this means that the diameter must exactly equal the side length of the square, ensuring that the circle touches all four sides. The same applies to a sphere within a cube, where the diameter must span the full height, width, and depth of the cube.

If a shape is smaller, off-center, or not fully enclosed in this way, the measured ratio will deviate and no longer correspond to the canonical GRM value. As such, both forward construction and reverse analysis must operate under the assumption that:

"The bounding form is the minimal square or cube that is fully contacted by the shape's outer boundary."

This constraint ensures that:

- GRM ratios remain mathematically valid,
- shape comparisons are consistent,
- and visual interpretations stay intuitive across design, education, and computation.

By preserving the logic of geometric enclosure, the GRM model maintains its internal coherence and proportional integrity.

4. Use Case: Ratio-Driven Shape Creation and Estimation

The GRM framework provides a flexible design logic that allows shapes to be either *constructed from intention* or *analyzed from observation*—all within a proportional, standardized container. This section presents a dual-use case: one for creating shapes with a precise target size, and one for estimating the properties of existing shapes based on GRM logic.

4.1 Creating a Shape Using the Hexagon Ratio

An illustrative example using a circle was already provided in section 2.1. To demonstrate the broader applicability of the GRM framework, we now consider a different shape: the regular hexagon.

Let us suppose a designer wants to create a regular hexagon with a target area of 17.32 cm^2. Using classical geometry, the area formula is: $A = \frac{3\sqrt{3}}{2}a^2$,
which requires solving for side length aaa and involves irrational roots—not ideal in many design environments.

The GRM-based approach reframes the problem:

1. A regular hexagon inscribed in a square occupies 0.8660 SAU.
2. To achieve an area of 17.32 cm^2, the square must have an area of:
$$A_{square} = \frac{17.32}{0.8660} \approx 20.00\ cm^2$$
3. The side length of the square is:
$$s = \sqrt{20} \approx 4.47\ cm$$
4. The hexagon is drawn inside this square, touching the appropriate edges.

This construction assumes the hexagon is perfectly inscribed within the square, in the canonical GRM configuration—resulting in the fixed 0.8660 ratio. Deviations from this configuration will yield different values.

This method:

- Simplifies design logic by removing irrational constants and intermediate steps,
- Provides a visual, ratio-driven approach to shape construction,
- Demonstrates the extensibility of GRM beyond circular forms.

4.2 Estimating the Area of a Detected Shape

Conversely, in digital imaging or design validation, a user may encounter a shape and wish to estimate its area based on how much of a square it fills.

For instance:

- A segmented object in a raster image is enclosed in a square bounding box.
- The object occupies approximately 78.5% of the square's area.

- This value closely matches 0.7854, the GRM ratio for an inscribed circle.
- The square has a side of 5 cm, so the bounding area is 25 cm^2.
- The object's estimated area is:

$$A_{object} = 0.7854 \times 25 = 19.64\ cm^2$$

The close match to 0.7854 suggests that the shape is a circle perfectly inscribed within the bounding square—fully touching all edges. This geometric condition is essential for the ratio to be valid and interpretable.

This approach:

- Offers an instant approximation of area or volume,
- Suggests the likely identity of the shape (in this case: a circle),
- Can be automated in software using simple ratio checks.

These dual pathways—from known ratios to shape, and from shape to inferred ratio—form the foundation of GRM's utility in proportional design, classification, and interactive geometry.

5. Mathematical Foundations: From SPU/SAU/SVU to Design Logic

The Geometric Ratio Model is based on fixed, dimensionless ratios that express how much of a bounding square (2D) or cube (3D) is occupied by a perfectly inscribed shape. These values serve not as approximations, but as structural constants for design, classification, and analysis.

5.1 Ratio Definitions within Standardized Forms

Within the GRM framework, the following standardized ratios are defined:**Shape**	**GRM Ratio**	**Relative Unit**	**Condition**
Circle	0.7854	SPU / SAU	Perfectly inscribed in square (diameter = side length)
Sphere	0.5236	SVU	Perfectly inscribed in cube (diameter = side length)
Regular Hexagon	0.8660	SPU / SAU	Inscribed in square with optimal orientation
Square or Cube	1.0000	SPU / SAU / SVU	Base unit for GRM system

These values are derived directly from classical geometry, but reframed as proportional standards rather than outcomes of irrational constants.

5.2 Rewriting Classical Formulas into GRM Logic

Instead of using formulas like

$$A = \pi r^2 \ or \ V = \frac{4}{3}\pi r^3$$

GRM recasts these into proportional expressions such as:

- $A_{circle} = 0.7854 \times A_{square}$
- $V_{sphere} = 0.5236 \times V_{cube}$

This transformation simplifies the process for both design and verification:

- No irrational numbers are needed during computation.
- Scaling and unit conversion become ratio-based.
- Visual logic becomes tightly coupled to metric reasoning.

5.3 Dimensional Consistency Across 1D, 2D, and 3D

A key strength of GRM is its dimensional coherence:

- In 1D, the perimeter of a square (4s) is the reference unit: 1 SPU.
- In 2D, the area of the square (s^2) becomes 1 SAU.
- In 3D, the volume of the cube (s^3) is 1 SVU.

Each added dimension builds on the same structural unit, enabling:

- Seamless cross-dimensional comparisons,
- Scalable modeling in CAD, design, and simulation environments,
- Consistent proportional interpretation across representations.

By grounding all calculations in a shared structural base (the square or cube), GRM unlocks a system where ratios replace formulas, and shapes become expressions of proportion. This makes geometry more accessible, programmable, and adaptable to both visual and computational workflows.

6. Application Examples and Use Potential

The GRM construction logic—anchored in fixed shape-to-container ratios—can be applied across a wide variety of disciplines. Its value lies in enabling consistent, scalable, and visually intuitive workflows for both digital and physical design tasks.

This section outlines representative examples from design, education, and digital tooling, illustrating the broad potential of ratio-based geometric construction.

6.1 CAD and Technical Drawing

In computer-aided design (CAD), precision and reproducibility are paramount. Traditional workflows require users to define shapes via dimensions or indirect geometric properties such as radius, diameter, or angle.

With GRM logic:

- Designers define a bounding square or cube based on a target size.
- The desired shape is drawn within this structure, automatically ensuring proper proportions.

- For example, a reservoir in an industrial component can be modeled as a sphere within a cube, using 0.5236 SVU to determine its fill ratio.

This method reduces error propagation, simplifies scaling, and supports modular component design across disciplines.

6.2 Educational Visualization Tools

For learners, geometry is often abstract and formula-driven. The GRM construction model transforms this into a hands-on visual framework:

- Students can physically or digitally draw a square and inscribe shapes within it.
- Using known ratios (e.g. 0.7854 for circles, 0.8660 for hexagons), they explore proportionality without needing π.
- The logic supports both discovery-based learning and conceptual reinforcement of area and volume.

Interactive tools (e.g., drawing software, geometry apps) can implement GRM logic to visualize construction ratios in real time.

6.3 Graphical Interface Layout and Design

In UI/UX design and animation, maintaining consistent visual ratios across elements is essential for balance, alignment, and responsiveness.

Using GRM:

- Designers can define interface components (e.g., buttons, icons, avatars) relative to bounding containers.
- Circular or hexagonal elements can be scaled based on area occupancy, ensuring proportional harmony regardless of screen size.
- Design systems can standardize component ratios using SAU/SPU constants, improving reusability and layout precision.

This introduces a logic of shape grammar by proportion, ideal for scalable design systems.

6.4 Modular Manufacturing and 3D Printing

In manufacturing contexts, especially in modular design and 3D printing, predictable sizing is critical. GRM-based construction enables:

- Defining components (e.g., spherical joints, hexagonal connectors) by bounding volume rather than internal dimensions.
- Maintaining consistency in scaling and fit, using SVU as a reference.
- Reducing computational load by replacing parametric recalculations with ratio-based templates.

This supports efficient iteration, component reuse, and consistent mechanical tolerances.

These examples illustrate how the GRM model extends beyond theory into practical design logic—enabling scalable construction, exact measurement, and intuitive visual feedback in both digital and physical domains.

7. Comparison with Classical Design Models

The GRM construction logic offers a fundamentally different approach to shape definition and measurement than classical geometry. While both methods are mathematically sound, their underlying logic, workflow efficiency, and conceptual structure differ in significant ways.

This chapter compares the two approaches across key dimensions relevant to design, education, and computation.

7.1 Conceptual Framework

Aspect	**Classical Geometry**	**GRM Construction Model**
Shape Definition	Based on internal dimensions (e.g., radius)	Based on external enclosure (bounding square/cube)
Use of Constants	Requires π and irrational roots	Uses fixed, rational ratios (SPU, SAU, SVU)
Design Entry Point	Requires formula derivation	Starts from proportional structure
Visual Logic	Abstract / calculation-driven	Visually grounded / container-first

GRM shifts the focus from *dimensionally-derived shapes* to *structurally-defined proportions*.

7.2 Practical Workflow Comparison

Workflow Step	**Classical Method**	**GRM Method**
Target: circle with area A	Compute radius via $r = \sqrt{\frac{A}{\pi}}$	Compute bounding square via $\frac{A}{0.7854}$
Tool implementation	Requires trigonometry or approximation	Requires square root and a fixed ratio
Tolerance control	Requires manual rounding	Ratio-based precision, visually aligned
Visual anchoring	Depends on correct radius scaling	Explicit through bounding structure

The GRM method simplifies construction and measurement, especially in visual or software environments where container-first logic aligns with raster, grid, or layout-based systems.

7.3 Interpretability and Educational Value

- Classical methods often abstract shapes into symbolic formulas, making them less accessible to early learners or non-mathematical users.
- GRM methods, by contrast, invite proportional reasoning, tangible construction, and intuitive understanding of dimensional relationships.

This makes GRM especially well-suited for:

- Didactic tools
- Shape libraries
- Parametric templates in CAD
- and AI shape interpretation modules.

7.4 Summary of Differences

Dimension	Classical Geometry	GRM Construction Logic
Mathematical Basis	Formulas and variables	Ratios and spatial units
Accessibility	Moderate to high barrier	Visually and computationally low
Flexibility	Rigid definitions	Scalable across dimensions
Interoperability	Context-dependent units	Unitless, proportional system

While classical geometry remains essential in analytical contexts, the GRM model provides a practical alternative in environments where visual reasoning, digital implementation, and structural proportionality are key.

8. Added Value and Implementation Possibilities

The GRM-based construction model offers distinct advantages across educational, technical, and computational domains. By unifying shape definition through fixed ratios within bounding forms, it provides a scalable and intuitive framework that enhances both human understanding and machine execution.

8.1 Practical Advantages of GRM Construction Logic

1. **Simplicity and Clarity**
    - Replaces irrational constants with fixed, human-readable ratios.
    - Reduces cognitive load in both learning and design workflows.
2. **Dimensional Consistency**
    - The same logic applies across 2D and 3D contexts (SPU, SAU, SVU).
    - Facilitates cross-dimensional modeling and comparison.
3. **Visual Anchoring**
    - Shapes are constructed or interpreted relative to a visible square or cube.
    - Improves communication between designers, engineers, and learners.
4. **Digital Readiness**

- Ideal for raster, grid, and voxel systems (e.g., CAD, UI, AI).
- Supports resolution-independent design and proportionally responsive layouts.

5. **Formula-Free Learning**
   - Promotes intuitive geometric understanding in early and remedial education.
   - Enables hands-on experimentation with spatial reasoning.

8.2 Implementation in Tools and Workflows

1. **Educational Geometry Apps**
   - Interactive platforms where users construct shapes within bounding squares.
   - Automatic feedback on proportions using GRM logic.
   - Integration with dynamic ratio sliders (e.g., target SAU = 0.7854).
2. **Design Software and CAD Plugins**
   - Ratio-based drawing modes, where users define the bounding structure first.
   - Templates for standard GRM figures (circle, hexagon, sphere).
   - Direct estimation of occupied volume or surface using SAU/SVU indicators.
3. **Programming and Scripting Libraries**
   - Open-source libraries in Python or JavaScript to generate or analyze GRM-based shapes.
   - GRM-aware geometry engines for UI layout, simulation, or procedural modeling.
4. **Physical Education Kits**
   - Laser-cut or 3D-printed square boards and shape cutouts.
   - Ratio labeling on each shape to reinforce visual proportion.
   - Classroom activities on volume filling, area estimation, and shape sorting.

8.3 Future Integration Paths

- **AI pipelines**: using GRM ratios as classifiers in geometric recognition tasks.
- **Responsive interfaces**: layouts driven by SAU/SPU logic for consistent UI scaling.
- **Parametric product design**: hardware parts defined by enclosure ratios instead of absolute sizes.

These directions align the GRM model with current trends in no-code design, data visualization, and STEM didactics—making it not just a theoretical innovation, but a practical system ready for integration.

9. Recommendations and Future Directions

The GRM-based construction framework introduces a shift in how shapes are conceptualized, defined, and built—offering a clear, scalable, and proportionally anchored alternative to classical geometry. To fully realize its potential, the following steps are recommended for future development, validation, and implementation.

9.1 Recommended Next Steps

1. **Develop Open Tools and Libraries**
   - Create open-source code libraries (e.g., in Python or JavaScript) to facilitate GRM-based shape construction and analysis.
   - Build a GUI-based prototype where users can draw within bounding forms and receive real-time ratio feedback.
2. **Integrate into Educational Platforms**
   - Implement GRM logic into interactive geometry tools for classrooms.
   - Develop lesson materials that teach ratio-based construction from primary education onward.
3. **Expand Shape Catalog**
   - Extend GRM ratios beyond circles, spheres, and hexagons to include ellipses, triangles, cylinders, and hybrid shapes—each with defined SAU/SVU equivalents.
4. **Build a Standardized GRM Vocabulary**

- Establish symbolic representations or icons for SPU, SAU, SVU in technical drawings and educational materials.
- Promote cross-disciplinary recognition of GRM ratios as legitimate units of geometric identity.

9.2 Long-Term Vision

The GRM model aligns naturally with the evolution of geometry toward visual reasoning, AI-based interpretation, and modular design systems. As more tools, platforms, and curricula adopt ratio-first thinking, GRM can serve as a bridge between:

- Classical theory and applied engineering,
- Human intuition and machine logic,
- Individual learning and system-wide standardization.

By positioning GRM as a universally applicable logic of geometric proportion, future developments may include:

- Shape validation layers in AI pipelines,
- Responsive UI layout engines based on SPU/SAU logic,
- A formal GRM metric system (parallel to SI) for scalable design thinking.

9.3 Final Remarks

This proposal has outlined the core reasoning, methodology, and applied value of constructing shapes within bounding forms using the GRM model. By replacing abstract formulas with concrete ratios, it enables a proportional design logic that is:

- Visually grounded,
- Dimensionally consistent,
- And ready for digital, educational, and physical deployment.

As the field of geometry continues to evolve alongside technology, GRM offers not just a new way to draw—but a new way to think about shape.

10. Conclusion

The GRM-based construction model offers a new geometric paradigm—one that replaces internal measurements and irrational constants with fixed, visually anchored ratios derived from bounding structures.

By defining shapes as proportional occupants of a square or cube, the model provides a scalable, intuitive, and dimensionally consistent framework for design, education, and analysis. Whether drawing a circle, estimating a hexagon's area, or verifying a shape in a digital system, GRM enables clarity through relational reasoning.

This proposal has outlined the theoretical foundation, practical logic, and diverse applications of GRM-driven shape construction. The strength of the model lies not only in its mathematical elegance but also in its adaptability: from classrooms to CAD systems, from paper to pixels.

In an increasingly digital, visual, and modular world, the GRM method introduces a universal language of proportion—ready to be applied, extended, and shared.